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THESIS

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SERRODYNING BY MEANS OF A DIGITAL PHASE SHIFTER

by

Meng-Yoon Tan

September 1989

Thesis Advisor: Jeffrey B. Knorr

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Serrodyning operation is employed by the velocity deception ECM jammer to counter those radars utilizing a target's doppler shift information. The jammer will frequency translate an incoming signal by a desired amount and at a specified rate during retransmission. If the radar follows the ECM signal, it is left without a target in its velocity gate when the ECM signal is turned off. The operation can be accomplished by either a TWT (Travelling Wave Tube) or a DPS (Digital Phase Shifter).

The purpose of this research is to determine the properties of the spectrum of a sinusoidal signal which is phase modulated with a staircase waveform. The investigations include the study of the qualities of the output spectrum and the characteristics of the operation, e.g., number of steps (phase resolution), phase accuracy, switching speed, translation loss and suppression ratio. Results are verified by simulations as well as by measurements.

The translation frequency, which is constrained by the switching speed of the device, affects the serrodyning performance the most. At higher translation frequencies, bits associated with small phase resolution do not contribute significantly to the suppression effect. The carrier frequency and input power have been found to have minimal effect on the serrodyning performance. Since the bandwidth of a radar's doppler filter is narrow, near side frequencies are less desirable in terms of degrading serrodyning effectiveness than those distant unwanted harmonic side frequencies, which may be of higher amplitude. The suppression ratio with reference to the nearest undesired side frequency is, in general, more than 25 dB. The original carrier residue has also been suppressed by at least 25 dB.

A DPS is better than a TWT in implementing serrodyning in terms of spectral purity, finite flyback, ease of implementation and controllability.

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Serrodyning By Means Of A Digital Phase Shifter

by

Meng-Yoon Tan
Captain, Republic of Singapore Air Force
B.Eng., National University of Singapore, 1983

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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ABSTRACT

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C.1

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I. INTRODUCTION

A. BACKGROUND

Some radars use the doppler shift of a target echo to track the target. The ECM technique used against such radars is known as velocity deception. It is implemented by receiving the radar signal at the target and then retransmitting it with a frequency that is slowly shifted. If the radar follows the ECM signal, it is left without a target in its velocity gate when the ECM signal is turned off. This technique is sometimes referred to as serrodyning (sawtooth modulation in Latin [Ref. 1]) or Velocity Gate Pull-Off (VGPO).

As shown in Figure 1a, with an input signal of $A \cos 2\pi f_c t$ being phase-modulated by another ramp signal $\theta(t)$ of frequency f_m, the resultant signal will be $A \cos 2\pi(f_c + f_m)t$. Having this in mind, the concept of serrodyning can be illustrated as in Figure 1b whereby a signal of frequency f_c is up-translated by a frequency of f_T. However, unless the translation is perfect, many undesired side frequencies will be produced along with the desired main component. Alternatively, down-translation will shift the output to $(f_c - f_m)t$.

Historically, the technique of serrodyning has been implemented employing an analog method using a TWT (Travelling Wave Tube) to accomplish the necessary phase modulation [Refs. 1,2]. The quality of the output signal spectrum depends upon the characteristics of both the TWT and the sawtooth waveform signal generator. In general, the undesired side frequencies can be suppressed by 20 dB or more relative to the desired output signal.

Recent advances in microwave component technology have resulted in the development of DPS (Digital Phase Shifter) which can also be used to obtain the same phase modulation function [Refs. 3, 4 and 5]. The quality of the output spectrum, in this case, depends upon the number of steps (phase resolution), the phase accuracy and the switching speed of the phase shifter.

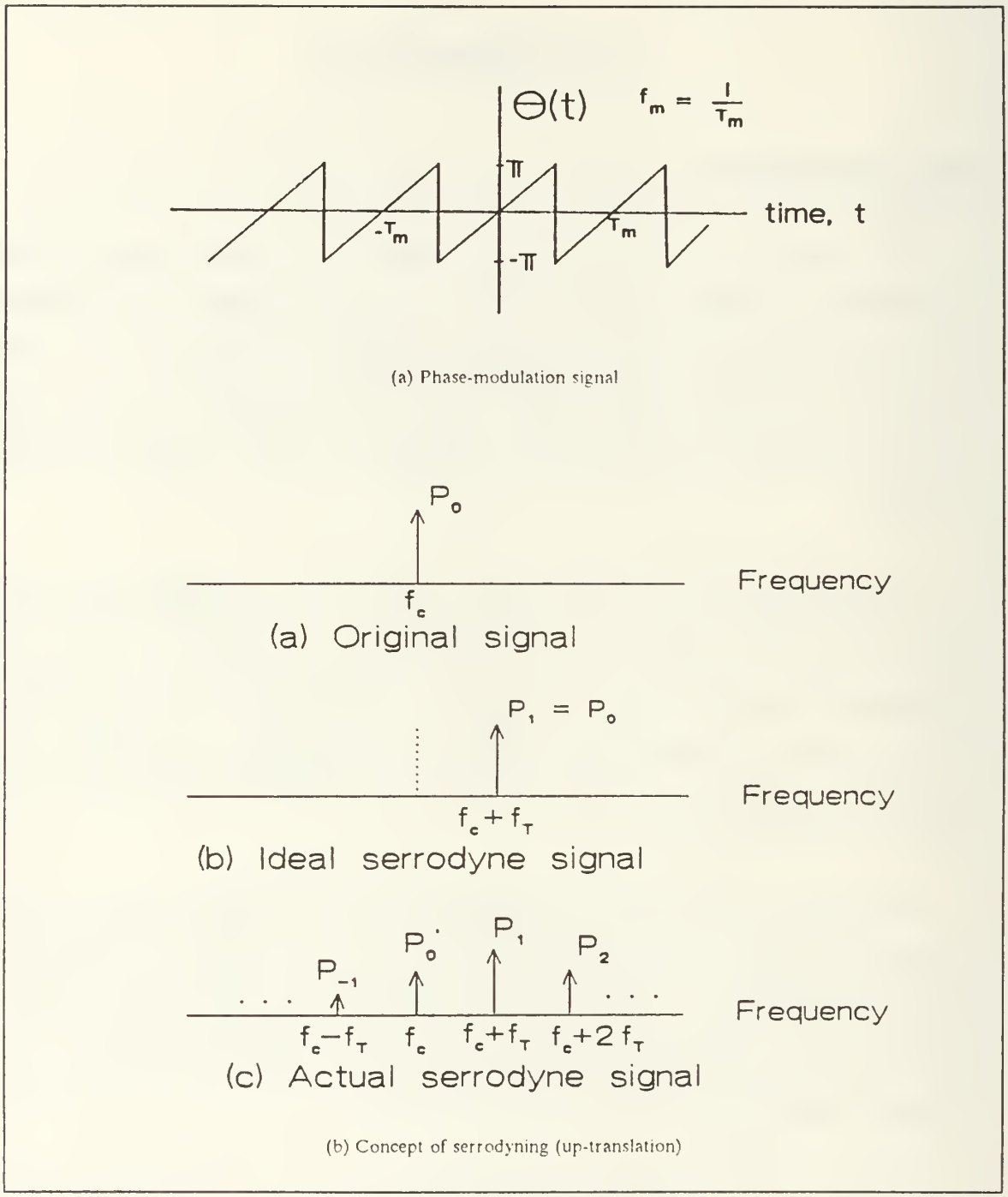


Figure 1. Serrodyning concept

B. PURPOSE

The purpose of this research is to determine the properties of the spectrum of a sinusoidal signal which is phase modulated with a step waveform using a DPS. Three approaches are pursued:

1. Theoretical analysis,

2. Simulation using HP Signal Simulator System, and

3. Laboratory measurement of a signal which is phase modulated using a DPS.

The results obtained from these three approaches are then analyzed and compared. This study provides definitive results on the relationship among spectral purity, switching speed, number of bits and the uniformity of step size of the staircase modulating waveform. Such information gathered is very useful in the design of velocity deception ECM using microwave digital phase shifters.

As far as the implementation is concerned, one would like to use the minimum circuit complexity necessary, in terms of the ease in implementation and controllability, to achieve the desired spectral purity.

C. REVIEW OF RELATED WORK/PAPERS

In Ref. 1, R.C. Cumming stated that when using a transit-time device, such as a TWT or Klystron, serrodyning could produce power output "practically equal to the capability of the same device operating as an ordinary amplifier", with very little undesired side frequency components (at least 20 dB suppression with respect to the main component). He also pointed out that any other modulable transit-time devices (for example, ferrite phase shifter) could be used for serrodyning. In Ref. 2, he reported that translation ranges from 0 to 60 MHz had been accomplished with TWT serrodyning and also provided a number of related design curves.

In Refs. 3 and 4, Rutz et al. and Jaffe et al. investigated the effect of approximating continuous sawtooth phase shift by a stepped phase shift in serrodyning. The specific

case of a 3-step approximation was also studied. G. Klein [Ref. 5] generalized the theory of an N-bit latching ferrite phase shifter "DIGILATOR" and described the design and testing of a 16-step, X-band device. Experiments showed that the performance of the serrodyning was independent of the microwave carrier frequency over the design frequency band and the suppression level of the carrier and the first side frequency was found to be as much as 39 dB.

Presently, some digital phase shifters still continue to be implemented by the switched-line type of selected length of lines, delay elements, or lumped-element low-pass and high-pass filters. The switching can be carried out by the use of FET or PIN diodes [Refs. 6,7,8,9]. Alternatively, a digital phase shifter may also be implemented by the vector modulator approach [Refs. 10,11,12], during which two quadrature signals are combined in a sine cosine ratio of a desired phase angle to produce a signal of constant amplitude with corresponding phase shift. The sine cosine ratio can be controlled by a digital attenuator.

II. SERRODYNING BY A DIGITAL PHASE SHIFTER

A. SERRODYNING IN GENERAL

Typical setups using a TWT and a DPS are depicted in Figure 2a and Figure 2b respectively. For the technique utilizing a TWT, the frequency shift that is required for the velocity deception is accomplished by phase modulating the received radar signal with a sawtooth waveform. It can be shown that if the sawtooth amplitude is adjusted to produce 2π radians of phase shift in T_m seconds, the power can be completely shifted to a frequency equal to the carrier frequency f_C plus the sawtooth frequency f_m. Digital phase shifters can also be used to obtain the same phase modulation function.

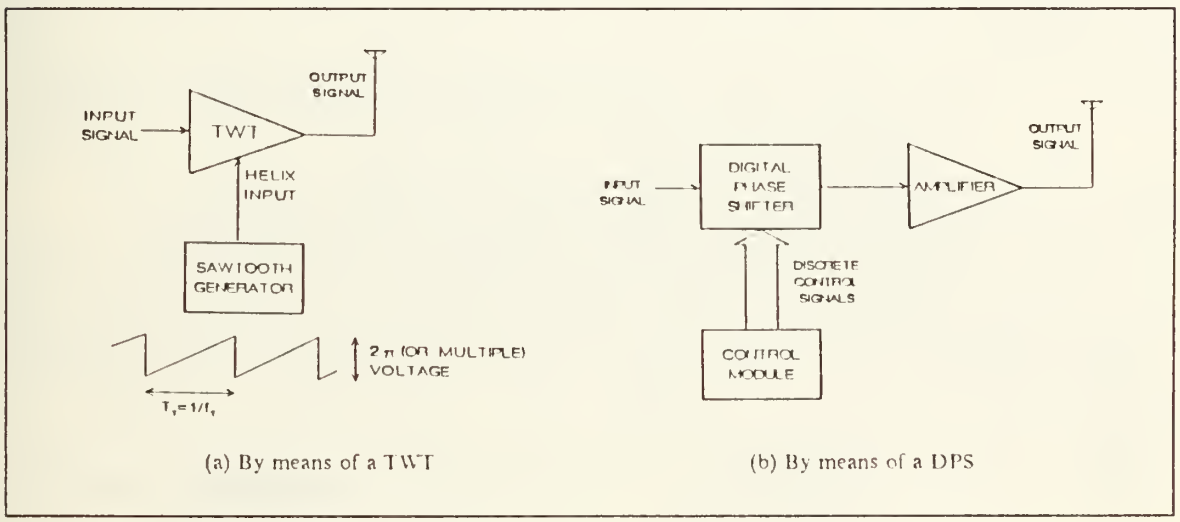


Figure 2. Serrodyning by means of a TWT or a DPS

B. DIGITAL PHASE SHIFTER

1. General Features

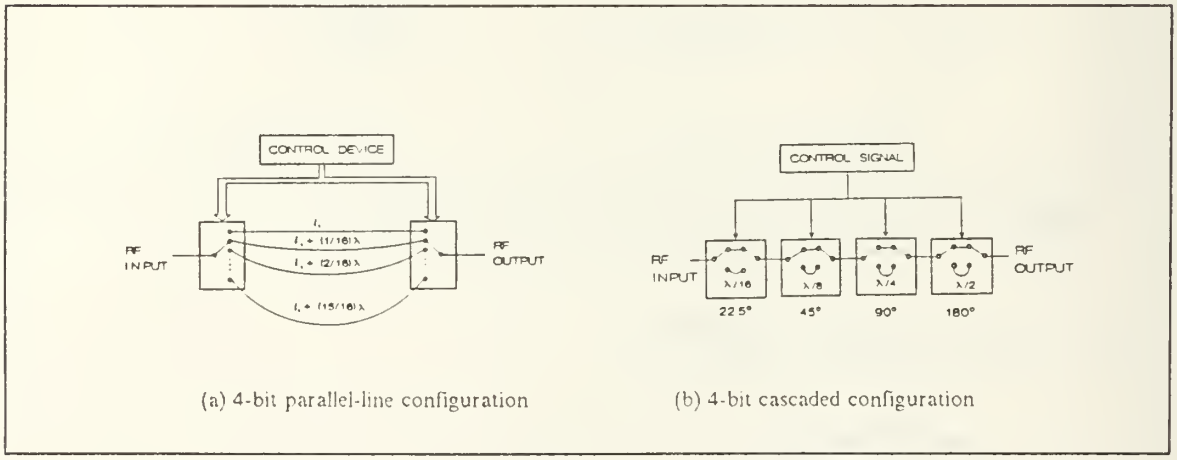
Phase shifters are widely used in various EW equipment. The desired features of a digital phase shifter are listed below:

- accurate phase change,
- rapid phase change (high switching speed),
- high phase resolution,
- broad operating frequency range,
- small value of insertion loss,
- consume low power for control signals,
- long operating life,
- small volume and low weight and
- low cost.

2. Types

There are many types of phase shifters, such as the waveguide phase shifter, the coaxial phase shifter, etc. As far as the digital phase shifter is concerned, mainly two types exist.

- Transmission Line Type
  - Parallel-line Configuration (See Figure 3a)
  - Series-line or Cascaded Configuration (See Figure 3b)
- RF Vector Modulation Type (See Figure 4)



(a) 4-bit parallel-line configuration

(b) 4-bit cascaded configuration

Figure 3. DPS of transmission line type

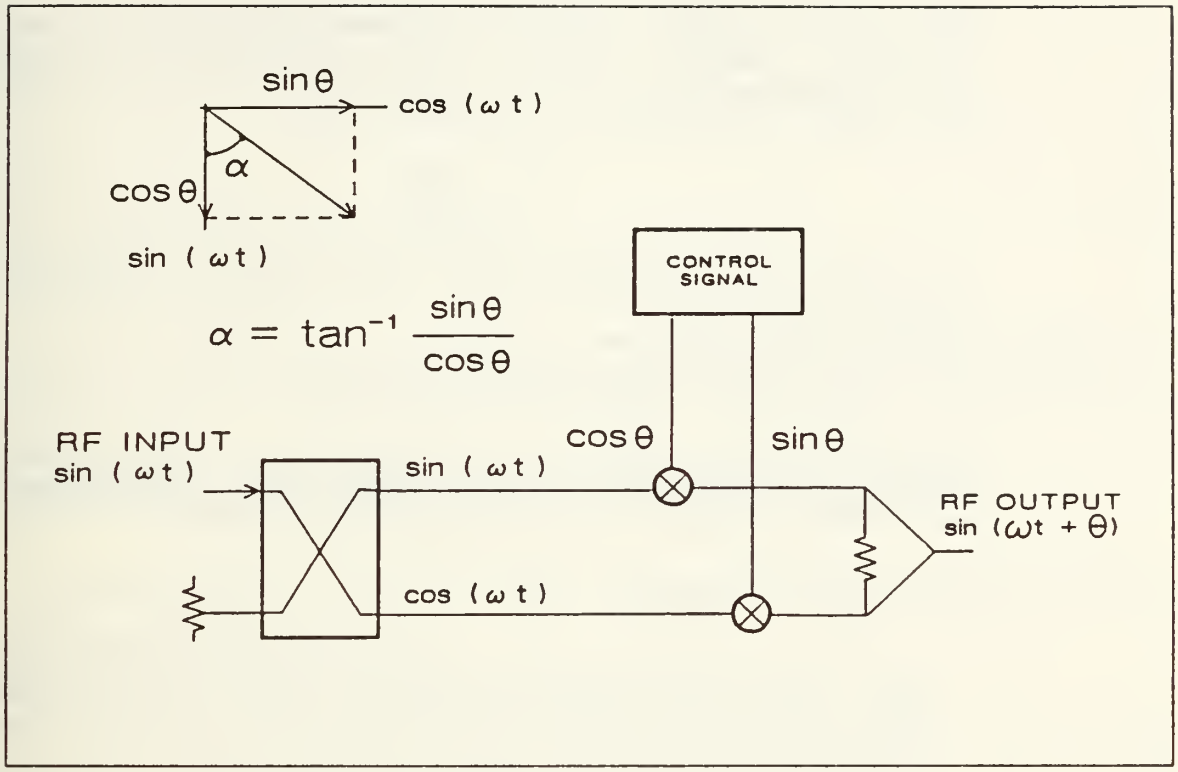


Figure 4. DPS of RF vector modulation type

The transmission line type simply switches a number of transmission lines of different lengths to achieve the desired phase shift. Reflection, loaded line or high/low-pass filter type may also be used to form phase shifts of various values. The RF vector modulation type, on the other hand, splits the input signal into two quadrature components with the 3-dB hybrid coupler. The two signals are individually modulated by the sine and cosine values of the corresponding phase shift. Finally, the modulated signals are added to give the phase-shifted output.

3. Characteristics

In general, the characteristics of a digital phase shifter can be described as follows:

- Phase Resolution

  The phase resolution is a very important parameter in the overall system performance. For instance, in the application of the phased-array radar,

phase shifters of higher resolution can better resolve the direction of the beam. In the serrodyne operation, it will determine the degree of perfection in frequency translation. The phase resolution $\Delta\theta$ depends on the number of bits available and employed. For a number B of bits, the phase resolution is given by

$$\Delta\theta = \frac{360^{o}}{2^{B}}. \qquad (2-1)$$

For example, if $B = 6$, $\Delta\theta$ will be 5.625^{o}. With the fabrication tolerance, $\Delta\theta$ should be around 5.6^{o}.

▲ Operating Frequency Range And Phase Accuracy

The phase accuracy is also a very important parameter. Generally speaking, operating frequency range affects phase accuracy which is also dependent on the component design as well as the manufacturing process.

△ Transmission Line Type

A narrow band phase shifter has a narrow designed operating frequency range. When it is used below this frequency, the phase shift accomplished is less than the design value. Above this frequency, the value of phase shift will be larger. A phase shifter with a wide operating frequency, on the other hand, requires a linear phase-shift characteristic, a built-in compensating network and a precise line length. In spite of these, phase inaccuracy value is still very large especially at the designed band edges.

△ RF Vector Modulation Type

In the absence of a transmission line, the phase accuracy depends mainly on the operating characteristics of the 3-dB hybrid coupler. As a result, this type of digital phase shifter can usually be made to operate over a wider bandwidth with good phase accuracy.

▲ Insertion Loss

Insertion loss is the reduction in input signal amplitude due to device characteristics. The reduction can normally be compensated by an amplifier. However, a larger value of insertion loss means a noiser system and uneven insertion loss will introduce unwanted amplitude modulation during signal processing.

▲ Switching Speed

Switching speed depends on the device design and will dictate how large the value of frequency translation can be.

III. THEORETICAL DERIVATION

A. MAJOR AREAS OF INTEREST

The static characteristics of a DPS and serrodyning using a DPS are of primary interest here. The static characteristics of the DPS are studied using scattering parameters.

B. STATIC CHARACTERISTICS (SCATTERING PARAMETERS)

Each individual phase state of a digital phase shifter can be considered as a 2-port network as modeled in Figure 5.

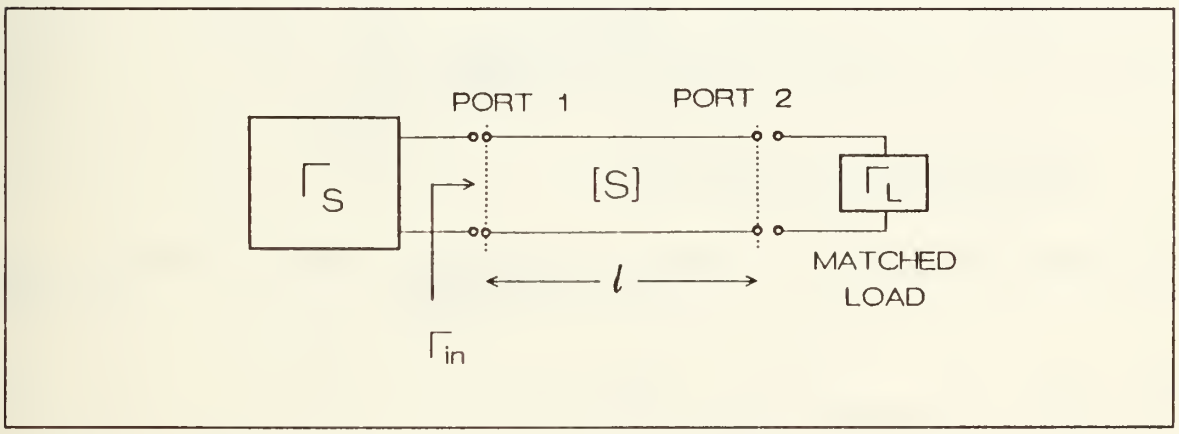


Figure 5. DPS as a 2-port network

Assuming that the network is symmetrical and reciprocal, the scattering parameters (or S-parameters) for the digital phase shifter are

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad (3-1.a)$$

$$= \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \qquad (3-1.b)$$

$$= \begin{bmatrix} 0 & 10^{-\frac{\alpha l}{20}} \angle -\beta l \\ 10^{-\frac{\alpha l}{20}} \angle -\beta l & 0 \end{bmatrix} \qquad (3-1.c)$$

where $\gamma = \frac{\ln 10}{20}\alpha + j\beta$, α is the loss in dB m and β is the phase lag in rad m.

The corresponding phase shift θ is

$$\theta = \beta l. \qquad (3-2)$$

The insertion loss I.L. and return loss R.L. are given by Eq. (3-3) and (3-4) respectively.

$$I.L. = -10 \log_{10} |S_{21}|^2 \qquad (3-3.a)$$

$$= \alpha l. \qquad (3-3.b)$$

$$R.L. = -10 \log_{10} |S_{11}|^2. \qquad (3-4)$$

The input reflection coefficient Γ_{in} is

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}. \qquad (3-5)$$

For a matched load, $\Gamma_{in} = S_{11}$.

C. SERRODYNING EFFECT

As the purpose of serrodyning is to perform frequency translation, only the fundamental frequency output is desired; other harmonic components are undesired. The performance of the translation can be described by the translation loss (T.L.) and the suppression ratio (S.R.). The following show the results and the detailed derivations are in Appendix A.

1. General Expression

Assume that a sinusoidal waveform $v_s(t)$ is being phase-modulated by a step function $\theta(t)$ as given by

$$v_s(t) = A \cos[\omega_c t + \theta(t)] \qquad (3-7.a)$$

$$= A\, Re\left[e^{j\omega_c t}\, e^{j\theta(t)} \right]. \qquad (3-7.b)$$

The phase $\theta(t)$ is a periodic positive rising ramp with frequency ω_m. The number of steps N is given by 2^B, where B is the number of bits being employed in serrodyning operation. Thus $v_s(t)$ can be represented by

$$v_s(t) = A \sum_{K=-\infty}^{\infty} C_K \cdot \cos(\omega_c + K\omega_m)t \qquad (3-8.a)$$

$$= A \sum_{\substack{K=-\infty \\ K=m\cdot N+1 \\ m=integer}}^{\infty} \frac{(-1)^m}{mN+1} \frac{\sin\frac{\pi}{N}}{\frac{\pi}{N}} \cdot \cos(\omega_c + K\omega_m)t. \qquad (3-8.b)$$

From Eq. (3-8.b), it is noted that the frequency of every non-zero component is given by

$$f = f_c + Kf_m \qquad (3-9.a)$$

$$= f_c + (mN+1)f_m. \qquad (3-9.b)$$

Examples of the values of C_K for various number of bits are given in Table 1. The values of C_K for 400 upper and lower harmonic terms can be found in Appendix B.

2. Expression For The Harmonic Spectral Components

The average power P_0 of an unserrodyned signal is given by Eq. (3-10). In the case of a serrodyne signal $v_s(t)$, the expression for average power P_K for corresponding K terms is given by Eq. (3-11).

Table 1. C CONSTANT FOR 1- TO 10-BIT

K	1-bit	2-bit	3-bit	4-bit	5-bit	6-bit	7-bit	8-bit	9-bit	10-bit
-14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-13	0.049	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-11	-.058	0.082	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-9	0.071	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-7	-.091	-.129	0.139	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-5	0.127	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-3	-.212	0.300	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-1	0.637	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.637	0.900	0.975	0.994	0.998	0.9996	0.9999	0.99997	0.99999	1.00000
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	-.212	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.127	-.180	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	-.091	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.071	0.100	-.103	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	-.058	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.049	-.069	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	-.042	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$$P_0 = \frac{A^2}{2}. \qquad (3-10)$$

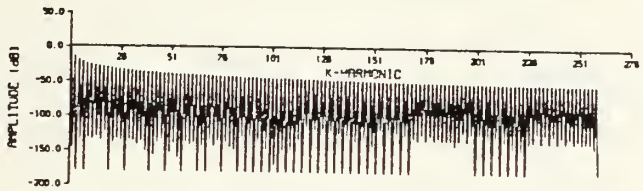
$$P_K = C_K^2 \frac{A^2}{2}. \qquad (3-11)$$

Spectral amplitude with respect to the unserrodyned signal is given by

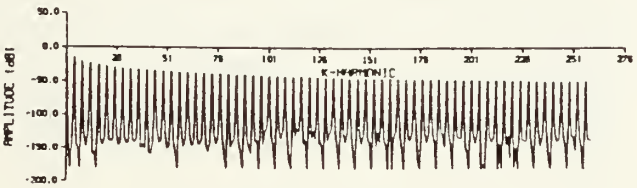
$$P_{K/0} = 10 \log \frac{P_K}{P_0} \qquad (3-12.a)$$

$$= 20 \log |C_K|. \qquad (3-12.a)$$

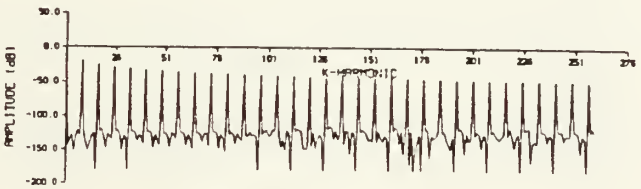
The spectral plots for 1- to 5-bit and 6- to 10-bit are in Figure 6 and Figure 7 respectively.



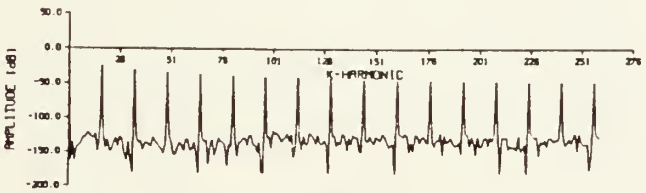
(a) 1-bit serrodyning



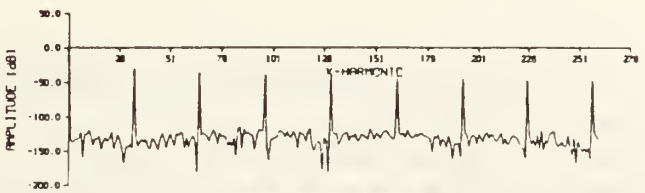
(a) 2-bit serrodyning



(a) 3-bit serrodyning

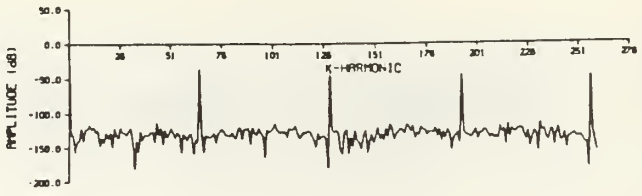


(a) 4-bit serrodyning

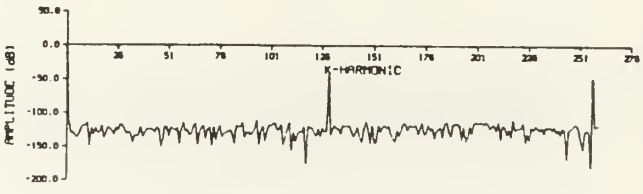


(a) 5-bit serrodyning

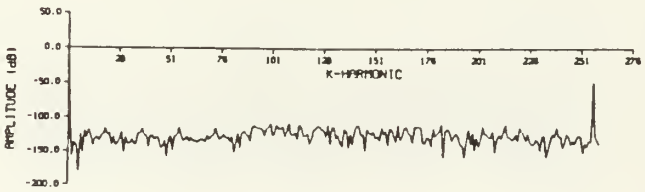
Figure 6. One-sided spectral plots for 1-bit to 5-bit serrodyning



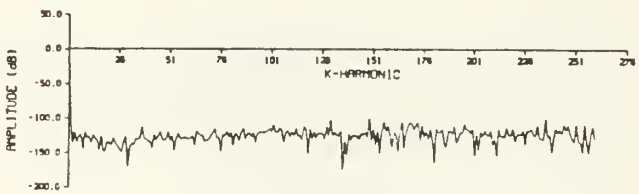
(a) 6-bit serrodyning



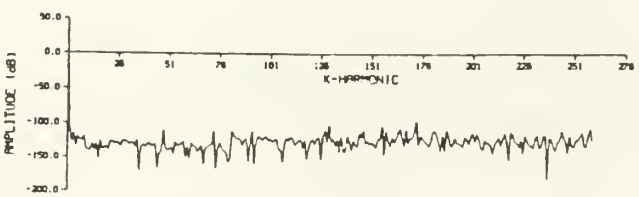
(a) 7-bit serrodyning



(a) 8-bit serrodyning



(a) 9-bit serrodyning



(a) 10-bit serrodyning

Figure 7. One-sided spectral plots for 6-bit to 10-bit serrodyning

The theoretical spectral plot consists of series of zero frequency linewidth components with the main component located at $f_c + f_T$ Hz.

Examples of values for $P_{K\cdot 0}$ for various number of bits are provided in Table 2. The values of $P_{K\cdot 0}$ for 400 upper and lower harmonic terms can be found in Appendix C.

Table 2. P CONSTANTS FOR 1- TO 10-BIT: In dB ratio.

K	1-bit	2-bit	3-bit	4-bit	5-bit	6-bit	7-bit	8-bit	9-bit	10-bit
-14	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-13	-26.2	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-12	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-11	-24.8	-21.7	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-10	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-9	-23.0	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-8	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-7	-20.8	-17.8	-17.1	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-6	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-5	-17.9	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-4	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-3	-13.5	-10.5	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-2	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
-1	-3.92	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
0	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
1	-3.92	-0.912	-0.224	-0.0559	-0.0140	-0.00349	-0.000872	-0.000218	-0.0000545	-0.0000136
2	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
3	-13.5	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
4	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
5	-17.9	-14.9	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
6	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
7	-20.8	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
8	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
9	-23.0	-20.0	-19.3	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
10	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
11	-24.8	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
12	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
13	-26.2	-23.2	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
14	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
15	-27.4	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$

3. Relative Amplitudes Of The Harmonic Components

It is essential to know the relative magnitude of each harmonic component. The fundamental component P_1 always has the largest magnitude. As for the harmonic components, the first lower harmonic P_{-N-1} is larger than the first upper harmonic P_{N-1}. This can be shown by evaluating the ratio of P_{-N-1} to P_{N-1}, which is given by

$$\frac{P_{-N+1}}{P_{N+1}} = \left(\frac{N+1}{N-1} \right)^2 . \qquad (3-13)$$

The ratio is always greater than 1 and approaches 1 as N gets larger.

4. Expression For Translation Loss (T.L.)

The translation loss of the serrodyning operation is defined as the power ratio of the unserrodyned signal over the fundamental component of the serrodyne signal [Ref. 1]. It describes the power loss of a signal due to the application of serrodyning effect. Figure 8 illustrates the definition of translation loss.

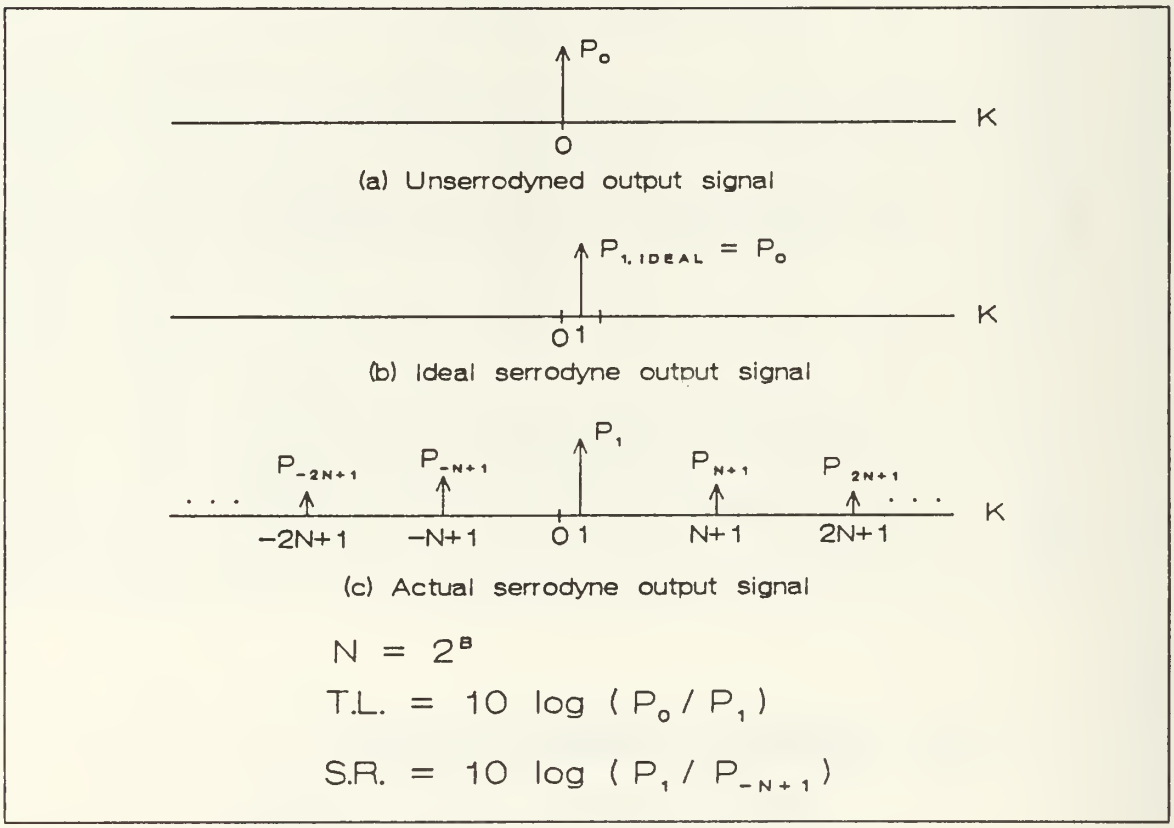


Figure 8. Definition of translation loss and suppression ratio

For a serrodyne signal, the expression for $v_1(t)$ and the average power of $K = 1$ component P_1 are given by Eq. (3-14) and (3-15) respectively. The translation loss can then be given by Eq. (3-16).

$$v_1(t) = \frac{\sin\frac{\pi}{N}}{\frac{\pi}{N}} A\cos(\omega_c + K\omega_m)t. \qquad (3-14)$$

$$P_1 = \left(\frac{\sin\frac{\pi}{N}}{\frac{\pi}{N}}\right)^2 \frac{A^2}{2}. \qquad (3-15)$$

$$T.L. = 10\log\frac{P_0}{P_1} \qquad (3-16.a)$$

$$= 20\log\left(\frac{\frac{\pi}{N}}{\sin\frac{\pi}{N}}\right). \qquad (3-16.b)$$

Figure 9 shows the plot of the theoretical translation loss (with the suppression ratio superimposed) versus the number of bits employed. Theoretically, T.L. only depends on the number of bits employed. The T.L. reduces very rapidly as the number of bits increases and exhibits the effect of diminishing marginal return as more bits are used. The T.L. is about 3.92 dB when 1 bit is used and reduces to near to 0 dB when 4 or more bits are used.

5. Expression For Suppression Ratio (S.R.)

The suppression ratio of the serrodyne signal is defined as the power ratio of the fundamental component over the first lower harmonic component. Figure 8 also explains the definition of suppression ratio. It is used to describe how well the serrodyning operation has suppressed the undesired harmonic components [Ref. 1]. In this case, the first lower harmonic component has the highest amplitude among all undesired components.

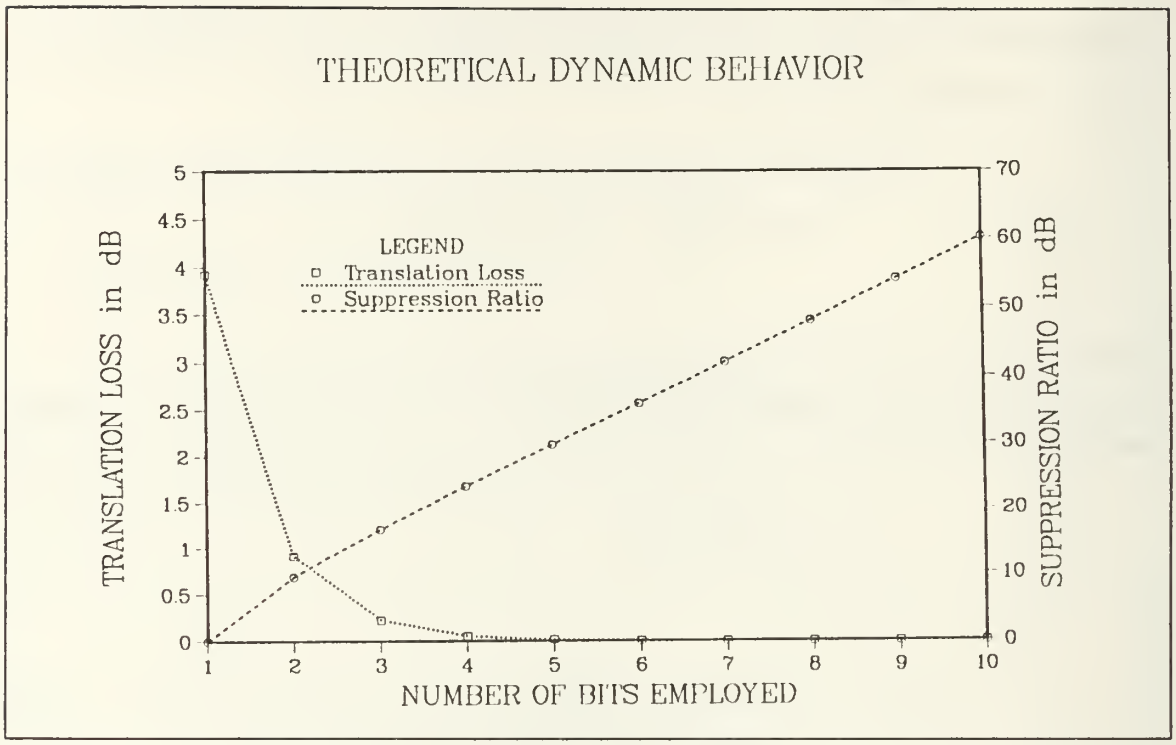


Figure 9. Theoretical plot for translation loss and suppression ratio

For a serrodyne signal $v_s(t)$, the harmonic component with the highest amplitude is given by $K = -N + 1$ term. The expression for $v_{-N+1}(t)$ and the average power P_{-N+1} are given by Eq. (3-17) and (3-18) respectively. The suppression ratio can then be derived as in Eq. (3-19).

$$v_{-N+1}(t) = \frac{-1}{-N+1} \frac{\sin \frac{\pi}{N}}{\frac{\pi}{N}} A \cos(\omega_c + K\omega_m)t. \qquad (3-17)$$

$$P_{-N+1} = \left(\frac{-1}{-N+1} \frac{\sin \frac{\pi}{N}}{\frac{\pi}{N}} \right)^2 \frac{A^2}{2}. \qquad (3-18)$$

$$S.R. = 10 \log \frac{P_1}{P_{-N+1}} \qquad (3-19.a)$$

$$= 20 \log (N - 1). \qquad (3-19.b)$$

Figure 9 also shows the plot of the theoretical suppression ratio (with the translation loss superimposed) against the number of bits employed. Theoretically, S.R. also only depends on the number of bits employed. The S.R. increases as the number of bits increases and also exhibits the effect of diminishing marginal return as more bits are used. The S.R. is 0 dB when 1 bit is used and is about 23.5 dB when 4 bits are used. However, when 10 bits are used, the S.R. can be as much as 60.2 dB.

6. Down-translation

In the derivations, up-translation is assumed. Nevertheless, theoretically, down-translation is expected to have the same serrodyning performance as the up-translation case except for the corresponding downwards frequency shift.

D. SUMMARY

- Spectral Plots in General

  When up-translation is performed, the main component is located at $f_c + f_T$ Hz. The lower side frequency has higher magnitude than the corresponding upper side frequency. The theoretical undesired side frequency of the highest amplitude is located at a multiple (2^B) of translation frequency away from the desired main component. Therefore, the frequency spacing between adjacent side frequencies doubles as the number of bits employed increases. The frequency linewidth of the serrodyne waveform does not increase with respect to the frequency linewidth of the incoming signal.

- Down-translation

  Down-translation has the same serrodyning performance as the up-translation case except for the corresponding downwards frequency shift.

- Relative Importance of Translation Loss and Suppression Ratio

  S.R. is more critical in serrodyning operation than T.L. since the latter can be compensated by an external amplification. Twenty dB of S.R. is considered sufficient in a general serrodyning opereation [Ref. 13].

- Effect of Number of Bits on Translation Loss and Suppression Ratio

  Theoretically, T.L. and S.R. only depend on the number of bits employed. T.L. reduces very rapidly as the number of bits increases. At the same time, S.R. improves. Both the T.L. and the S.R. exhibit the effect of diminishing marginal

return as more bits are used. The T.L. is close to 0 dB when 4 or more bits are used. As for the S.R., it is about 23.5 dB when 4 bits are used. However, when 10 bits are used, the S.R. can be as much as 60.2 dB.

IV. SIMULATION FOR SERRODYNING EFFECT

A. EQUIPMENT IN GENERAL

Figure 10 shows the simulation setup used to verify the serrodyning effect of a DPS. The HP8770S Signal Simulator System [Refs. 14,15,16] comprises an HP8770A Arbitrary Waveform Synthesizer (HP AWS) and the HP11776A Waveform Generation Language Software running on a HP9000 series 200/300 computer. The system is capable of generating a complex baseband waveform with frequency from dc to 50 MHz in either the frequency or time domain. In addition, the baseband signal can also be upconverted to a higher frequency band. The spectrum of the output signal can then be studied using a Spectrum Analyzer.

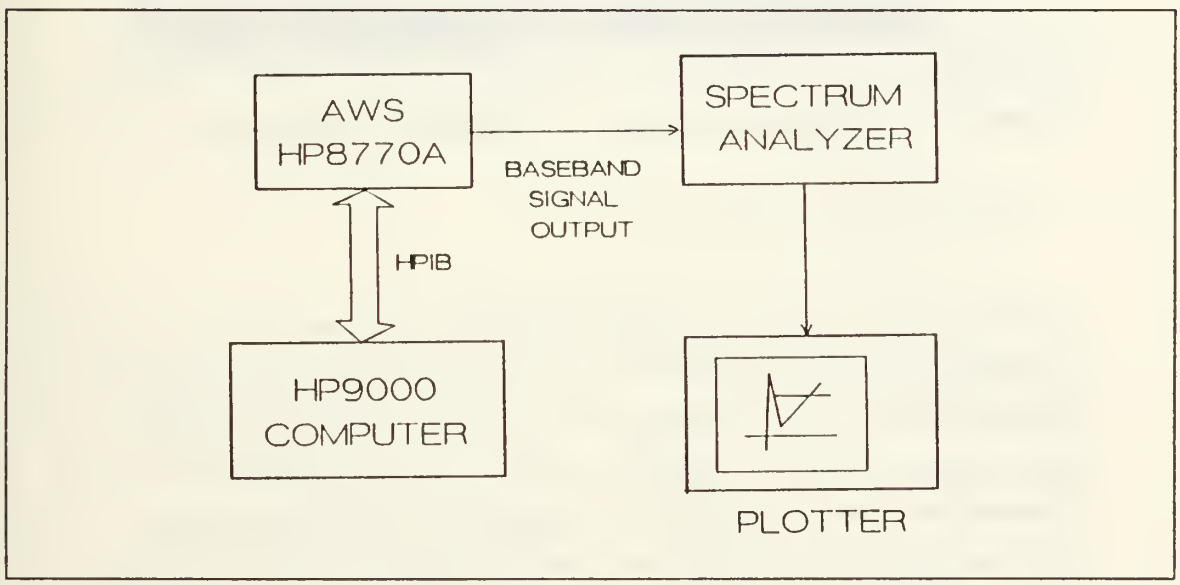


Figure 10. Simulation setup using HP Signal Simulator System

The required waveform is generated in the computer and then downloaded to the synthesizer as digital information. The digital synthesizer has a capability of 12-bit resolution (72 dB dynamic range), a 512 kbyte high speed memory and a 125 MHz (samples per second) clock. A DAC output stage is used to generate the signal. The RF

power output is more than 0 dBm. Figure 11 shows an example of the serrodyne waveform generated by the system software.

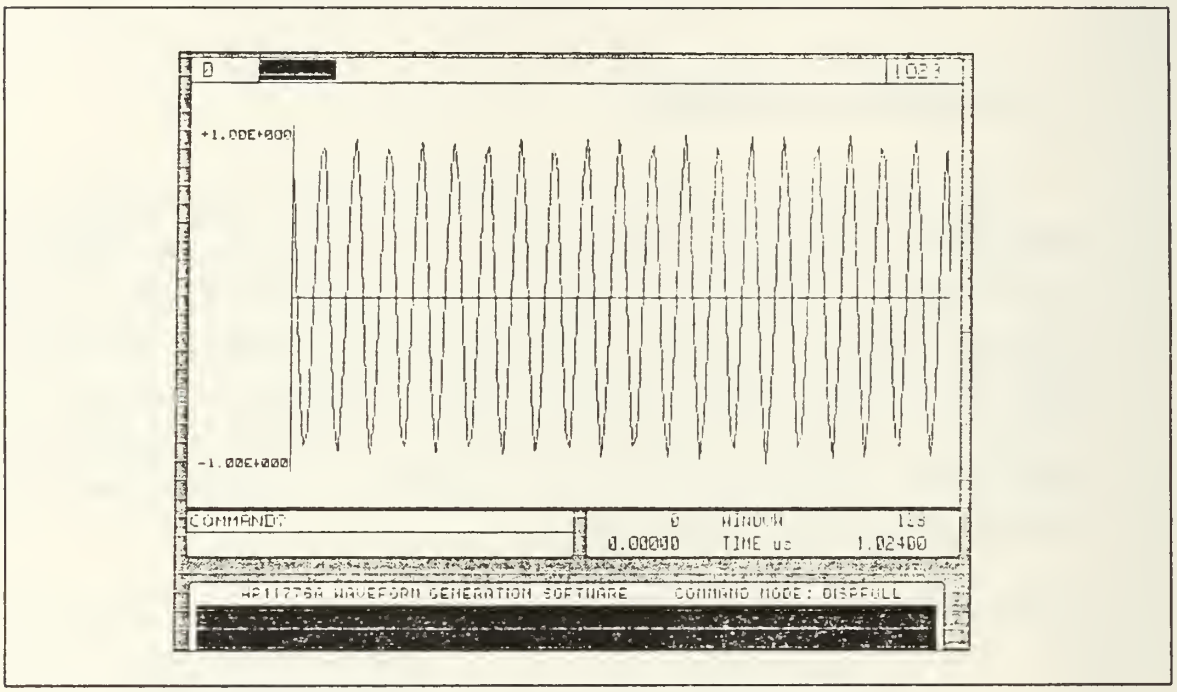


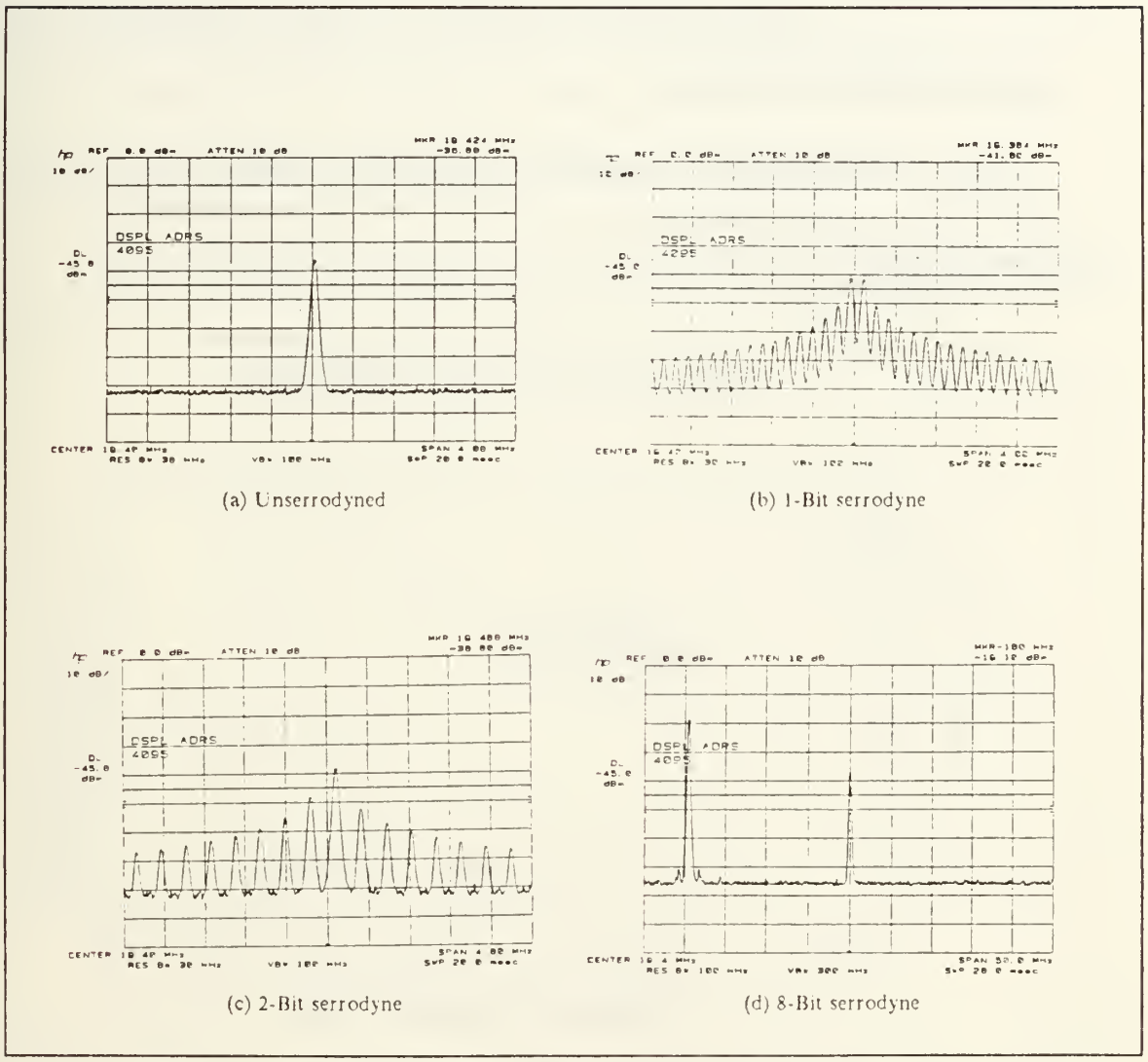
Figure 11. Example of a serrodyne waveform (by screen dump)

B. SIMULATION APPROACH

For verification purposes, a baseband phase-modulated waveform is generated using a simple utility program (reproduced in Appendix D) written in Waveform Generation Language (WGL) [Ref. 17]. Since the waveform generation is periodic, the sinusoid must complete one or more additional complete cycle transitions in T_m seconds. Due to the limitation of the equipment clock frequency as well as the memory available, the carrier and translation frequency are bounded within a relatively small range of values. Table 3 presents one of the many test combinations used.

Table 3. SIMULATION COMBINATIONS

Category	Values
Carrier frequency	19.424 MHz
Translation frequency	0.064 MHz
Translated frequency	19.488 MHz
Bit	1 to 10



(a) Unserrodyned

(b) 1-Bit serrodyne

(c) 2-Bit serrodyne

(d) 8-Bit serrodyne

Figure 12. Spectral plots for unserrodyned and 1-bit, 2-bit and 8-bit serrodyne: The frequency span used in (d) is 50 MHz instead of 4 MHz as in (a) to (c).

C. SIMULATION RESULTS

Figure 12a to Figure 12d are the spectral plots for the unserrodyned, as well as the 1-bit, 2-bit and 8-bit serrodyne cases. Other spectral plots are given in Appendix D.

The noise floor was not raised noticeably due to serrodyning operation. However, since the difference between the noise floor and the peak carrier power is only about 50 dB in our case, the S.R. cannot be measured for $B \geq 9$ bits. The linewidth gets only marginally broader in this case.

Figure 13 is the plot of the translation loss and suppression ratio measured under this simulated condition (theoretical T.L. and S.R. are also superimposed here for comparison purpose).

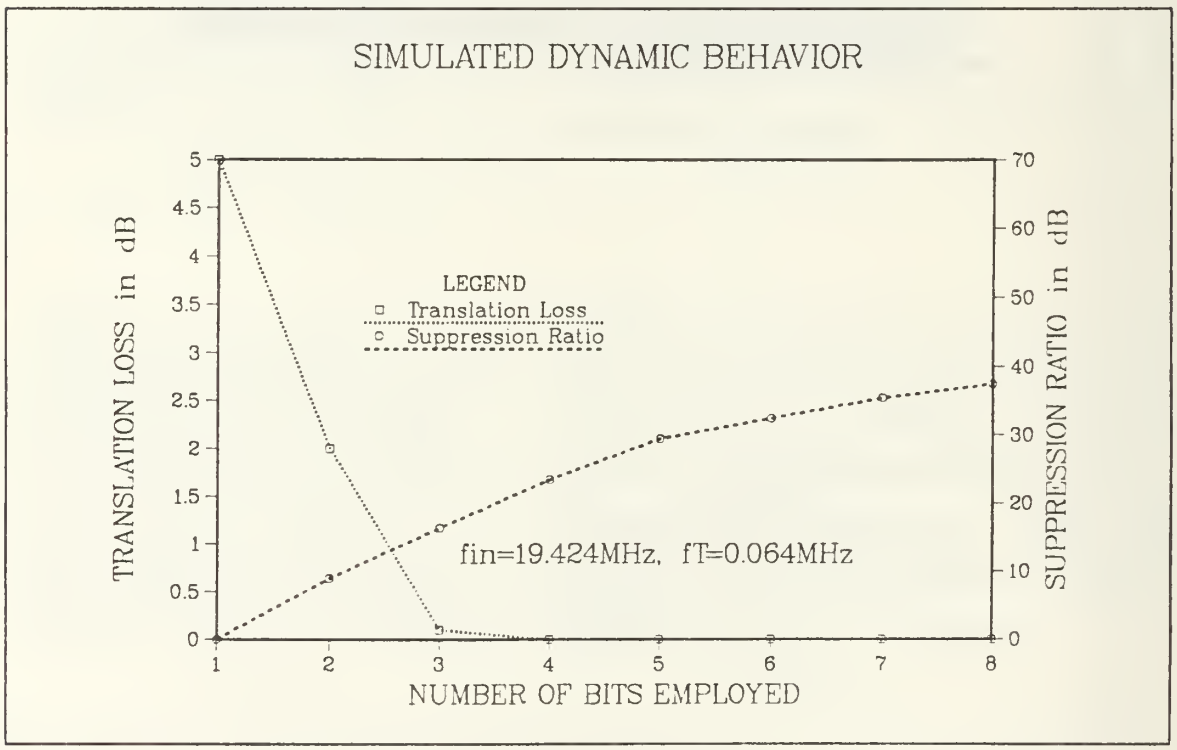


Figure 13. Plot for translation loss and suppression ratio by simulation

The results obtained under simulation correlate very well with the theoretical values. However, the suppression ratio is lower than the respective theoretical result. For example, the S.R. is 32.4 dB in this case compared to 36.0 dB theoretically. As for the translation loss, it is higher at lower number of bits as compared with the theoretical result.

D. SUMMARY

- The spectral plots are the same as the theoretical ones.
- The noise floor was not raised noticeably due to the serrodyning operation.
- The linewidth gets only marginally broader in the simulated case.
- The results obtained under simulation correlate rather well with the theoretical values. However, the suppression ratio is slightly less than the theoretical value.

V. LABORATORY MEASUREMENT

A. GENERAL INFORMATION

1. Technical Details Of The Digital Phase Shifter Used

A *Qual – Tech*[1] 6-bit digital phase shifter has been used to validate the formulae and the simulation results. The essential technical data for this component are shown in Table 4 [Ref. 18]. Its internal implementation is of a cascaded configuration with pin diodes and delay elements (see Figure 14).

Table 4. TECHNICAL DATA OF THE DPS

Part Number	DP-6021
Description	Digital Diode Phase Shifter
Frequency range	10.0 to 10.5 GHz (maximum)
Number of bits	1 to 6
Least Significant Bit	5.6^{o}
Phase accuracy	$\pm 3^{o}$ (typ) / 8^{o} (max)
Insertion loss	8.5 dB (maximum)
V.S.W.R.	1.8 : 1 (maximum)
Switching speed	2 μs
Control logic	TTL

2. Types Of Measurement

Two types of measurement have been carried out. They are:

- Static characteristics (Scattering-parameter measurement) and
- Serrodyning effect.

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1 Trade Mark of Quality Technology.

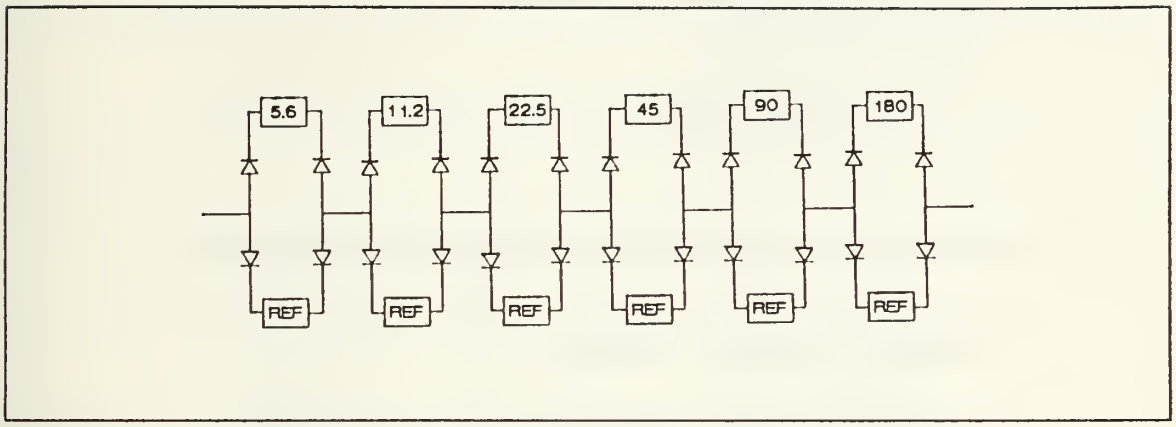


Figure 14. Cascaded configuration

3. Experiment Setup

The experimental setup for both static characteristics and serrodyning effect measurements are shown in Figure 15a and Figure 15b respectively.

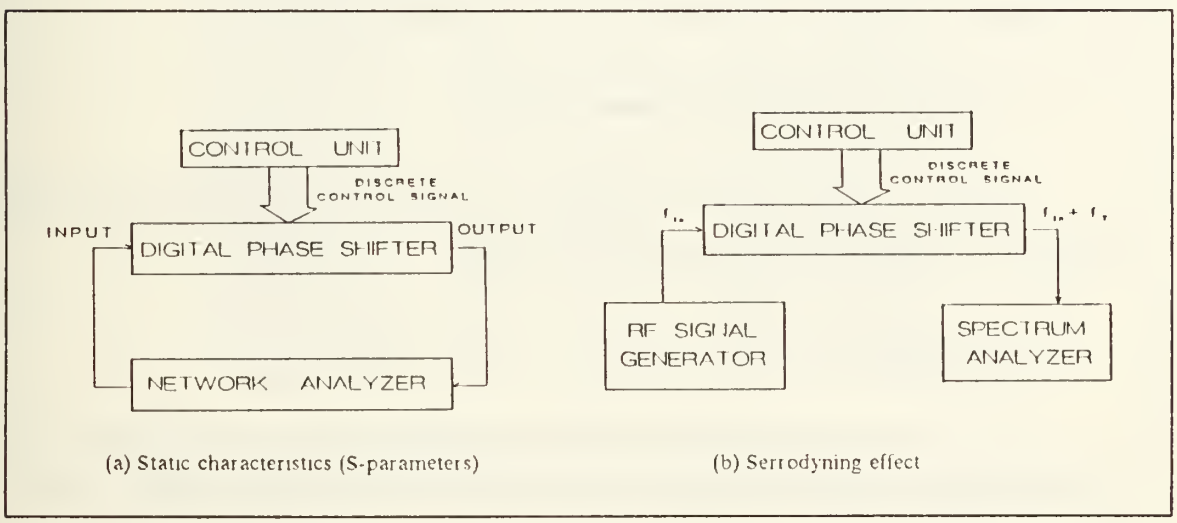


Figure 15. Setup for measurements

A control circuit with the following features has been fabricated (for details, see Appendix D):

- a switch to control whether static characteristics or serrodyning effects are to be measured;
- a switch to select up- or down- translation;

- a 6-bit binary counter to generate TTL control logic signals to drive the digital phase shifter;
- a 6-bit switch to enable disable a particular phase bit; and
- a switch to reset all bits.

B. STATIC CHARACTERISTICS (SCATTERING PARAMETERS)

1. Equipment Description And Results

The S-parameters can be obtained using the HP Vector Network Analyzer (HP8409B 0.11-18 GHz) while controlling the individual discrete phase shifts. An HP11866A APC-7 Calibration Kit is used to calibrate the system before as well as after the S-parameter measurements. An 8-term error model is used.

The S_{11} and S_{21} parameters for each of the 64 phase states have been measured. For example, Figure 16 lists the printout of the measurement for the phase state 11.25^{o} and Figure 17a to Figure 17d show the corresponding plots.

2. Linearity In Phase Shift Characteristics

Based on the plot of S_{21}-DEG shown in Figure 17c, it is observed that the phase shift characteristic of the component is fairly linear.

3. Return Loss

Since the maximum VSWR of this digital phase shifter is specified to be less than or equal to 1.8:1, the reflection coefficient can be calculated as follows:

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1} \qquad (5-1.a)$$

$$\leq 0.2857. \qquad (5-1.b)$$

For a matched load, the return loss will be :

$$R.L. = -10 \log_{10} |S_{11}|^2 \qquad (5-2.a)$$

DPS-111101,11.25,

FREQUENCY	RETURN LOSS-IN S11		LOSS-FORWARD S21	
MHz	DB	ANG	DB	ANG
9500.0000	12.36	151.5	8.03	54.4
9550.0000	12.10	134.0	8.30	30.1
9600.0000	12.19	112.2	8.31	7.3
9650.0000	12.54	94.3	8.35	-16.3
9700.0000	12.24	74.6	8.60	-39.8
9750.0000	12.78	52.8	8.60	-62.0
9800.0000	13.12	36.6	8.58	-86.0
9850.0000	13.18	15.5	8.80	-108.2
9900.0000	14.14	-4.2	8.66	-130.4
9950.0000	14.54	-21.5	8.70	-154.4
10000.0000	15.18	-44.2	8.75	-177.1
10050.0000	16.57	-66.5	8.65	159.3
10100.0000	17.80	-90.4	8.70	135.3
10150.0000	19.76	-122.9	8.65	112.5
10200.0000	23.14	-160.2	8.54	89.1
10250.0000	24.59	153.7	8.55	65.5
10300.0000	23.08	91.7	8.45	50.3
10350.0000	21.48	43.2	8.34	18.1
10400.0000	19.33	13.8	8.35	-6.4
10450.0000	17.24	-18.4	8.30	-30.3
10500.0000	16.54	-47.2	8.25	-55.0
10550.0000	15.81	-71.2	8.35	-79.7
10600.0000	15.39	-98.4	8.45	-103.8
10650.0000	15.67	-122.7	8.50	-127.7
10700.0000	15.40	-146.0	8.63	-152.4
10750.0000	15.66	-173.9	8.85	-176.4
10800.0000	16.44	163.2	8.90	159.7
10850.0000	16.54	138.9	9.04	133.8
10900.0000	17.59	112.3	9.35	110.2
10950.0000	18.67	91.4	9.35	86.6
11000.0000	19.78	66.7	9.64	62.0

Figure 16. Printout for the S-parameter measurement: Phase state of 11.25°.

$$= -10\log_{10}|\Gamma|^2 \qquad (5-2.b)$$

$$\geq +10.9\ \text{dB}. \qquad (5-2.c)$$

From the plot of S_{11}-dB shown in Figure 17b, the reflection loss has been found to be at least 12 dB, which is better than the manufacturer's specification.

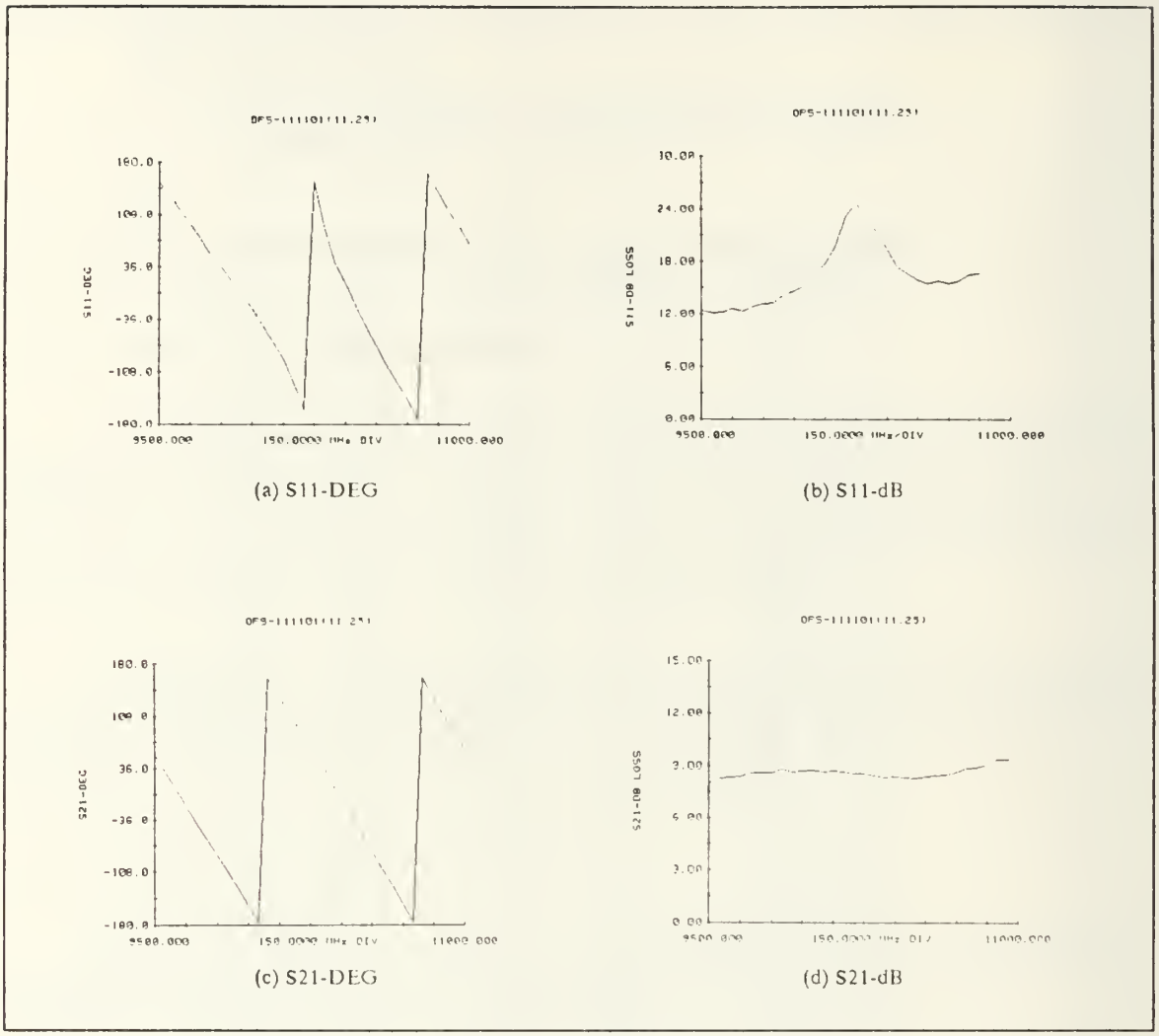


Figure 17. Plots for S-parameter measurement: S_{11} and S_{21} plots for the phase state 11.25°.

4. Insertion Loss

Furthermore, to study the characteristics of the device at 10.25 GHz (mid-frequency of the device operating range), the respective insertion loss and the relative phase (ΔAng) of each phase state at this frequency respectively are plotted as Figure 18a and Figure 18b (details are tabulated in Appendix E)

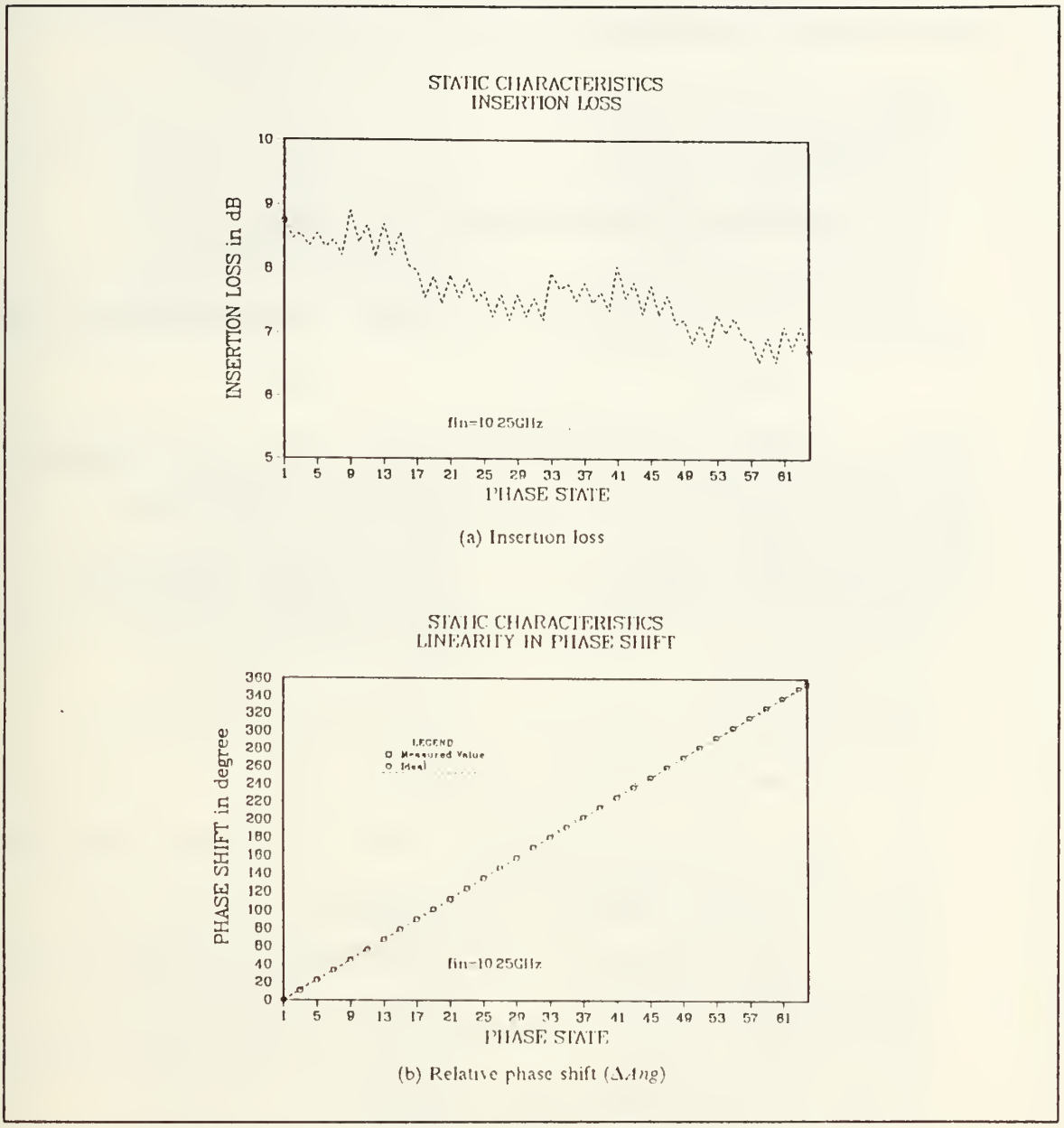


Figure 18. Insertion loss and relative phase shift: At carrier frequency of 10.25 GHz.

Insertion loss has been found to be about 7.64 dB, which is the averaged S_{21}(dB) value for the frequency of 10.25 GHz. This value is better than the 8.5 dB specified by the manufacturer. Generally, uneven insertion loss with respect to individual phase state

in the static characteristics study (from Figure 18a) indicates the presence of an inherent undesired amplitude modulation effect during serrodyning operation.

C. SERRODYNING EFFECT

1. Combinations Of Parameters Used In Measurements

Table 5 lists the combinations of the various parameters used for the measurement.

Table 5. COMBINATIONS OF PARAMETERS USED IN MEASUREMENT

Carrier frequency, f_{in} (GHz)	9.5, 10.0, 10.25, 10.5, 11.0
Input power, P_{in} (dBm)	-40, -30, -20, -10, 0
Translation frequency, f_T (kHz)	-10, 0, 2, 4, 6, 8, 10, 20, 40, 80, 160
Number B of bits	0, 1, 2, 3, 4, 5 and 6

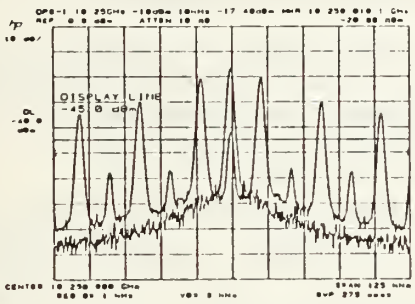
2. Measurements

1) Basic Serrodyning Effect

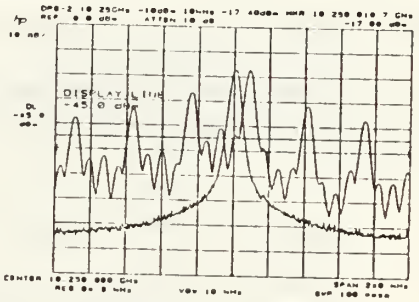
Based on the parameters given in Table 5, the following base set has been chosen for comparison purpose:

- Number B of bits = 1, 2, 3, 4, 5 and 6
- Carrier frequency, f_{in} = 10.25 GHz (mid-frequency of the device operating range)
- Input power, P_{in} = -10 dBm (measured at the output of the signal generator)
- Translation frequency, f_T = +10 kHz

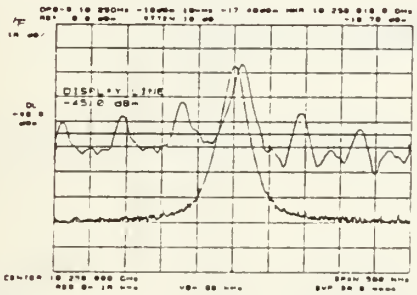
The spectral plots for the 1-bit to 6-bit cases are shown in Figure 19a to Figure 19f respectively. The experimental data for the measurements are shown in Appendix E. The linewidth gets broader marginally in the actual measurement. The noise floor is also raised substantially due to serrodyning operation.



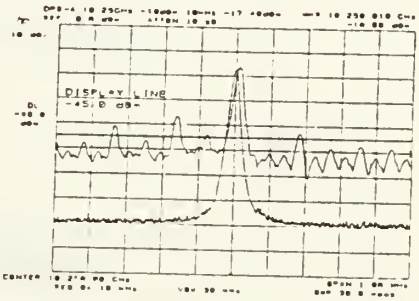
(a) 1-bit



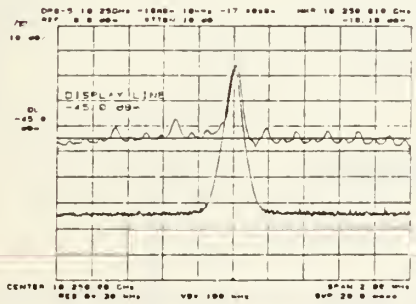
(b) 2-bit



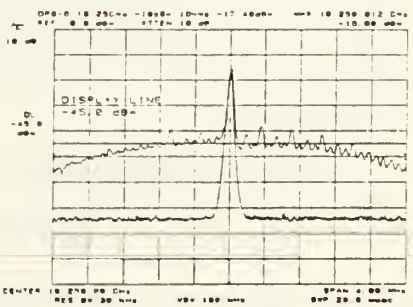
(c) 3-bit



(d) 4-bit



(e) 5-bit



(f) 6-bit

Figure 19. Spectral plots for 1-bit to 6-bit serrodyne operation: The top and bottom traces are the spectrums of the serrodyne and unserrodyned signals respectively. The unserrodyned spectrum is obtained by resetting all the control bits to inactive states (i.e., '1's in this case). The frequency span is doubled consecutively from (a) to (f) so that each spectrum can contain at least four undesired side frequencies.

The corresponding plots for T.L. and S.R. are shown in Figure 20. Similar to the theoretical results, the translation loss reduces as the number of bits increases; correspondingly, the suppression ratio improves.

However, in this case, the S.R. is not as good as those predicted theoretically. Though the S.R. is about 20.3 dB (23.5 dB theoretically and 23.4 dB by simulation) when 4 bits are used, it is saturated to about 22.1 dB (36.0 dB theoretically and 32.4 dB by simulation) when 6 bits are used.

The T.L. is lower than both the corresponding theoretical and simulated ones and even becomes negative for 3 or more bits. It is possible that the T.L. becomes negative during serrodyning operation because an unserrodyned signal is obtained by resetting all the phase bits to in-active states (corresponding to 0^{o} state). The insertion loss at 0^{o} state is 8.75 dB while the average insertion loss is 7.64 dB.

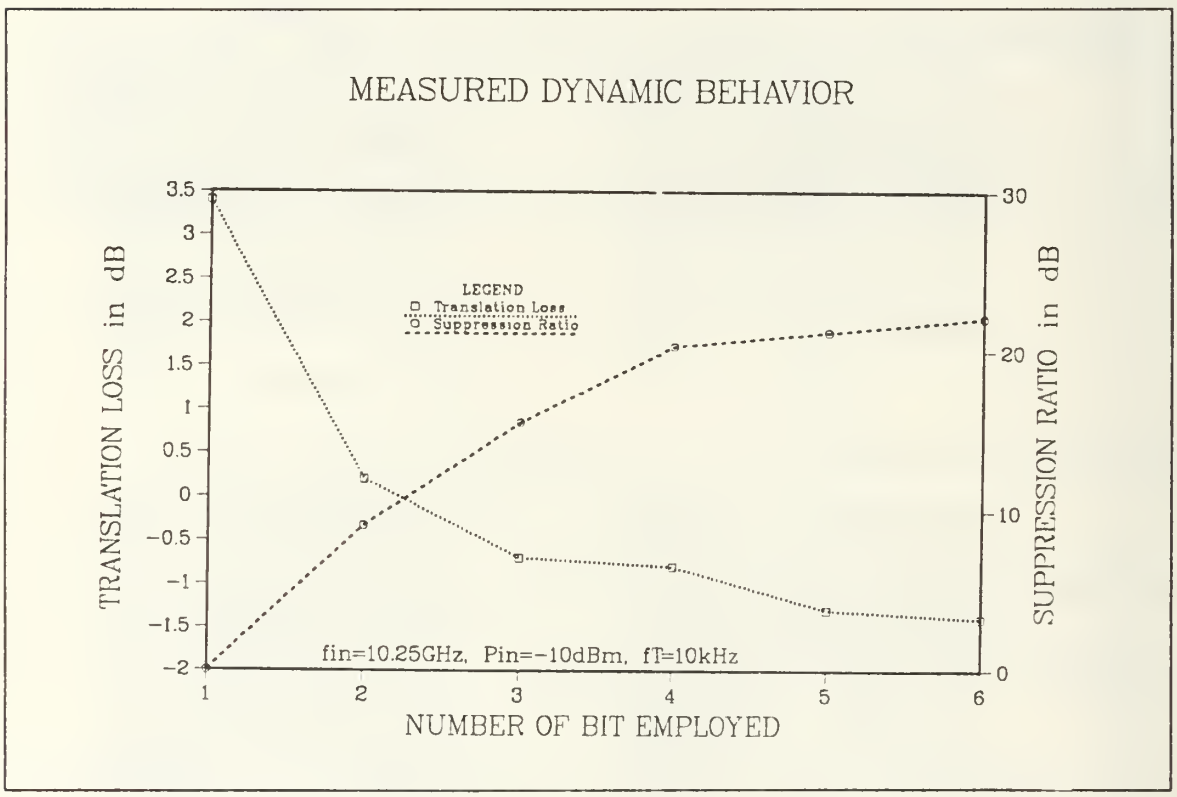


Figure 20. Basic serrodyning effect (T.L. and S.R.)

2) Others

The following effects have also been studied:

- Effect of down-translation frequency on T.L. and S.R. (See Figure 21).

It is important to ascertain that both up- or down-translation will have the same serrodyning performance. From Figure 21, down-translation produces a signal of the corresponding down-shift with essentially the same amount of T.L. and S.R..

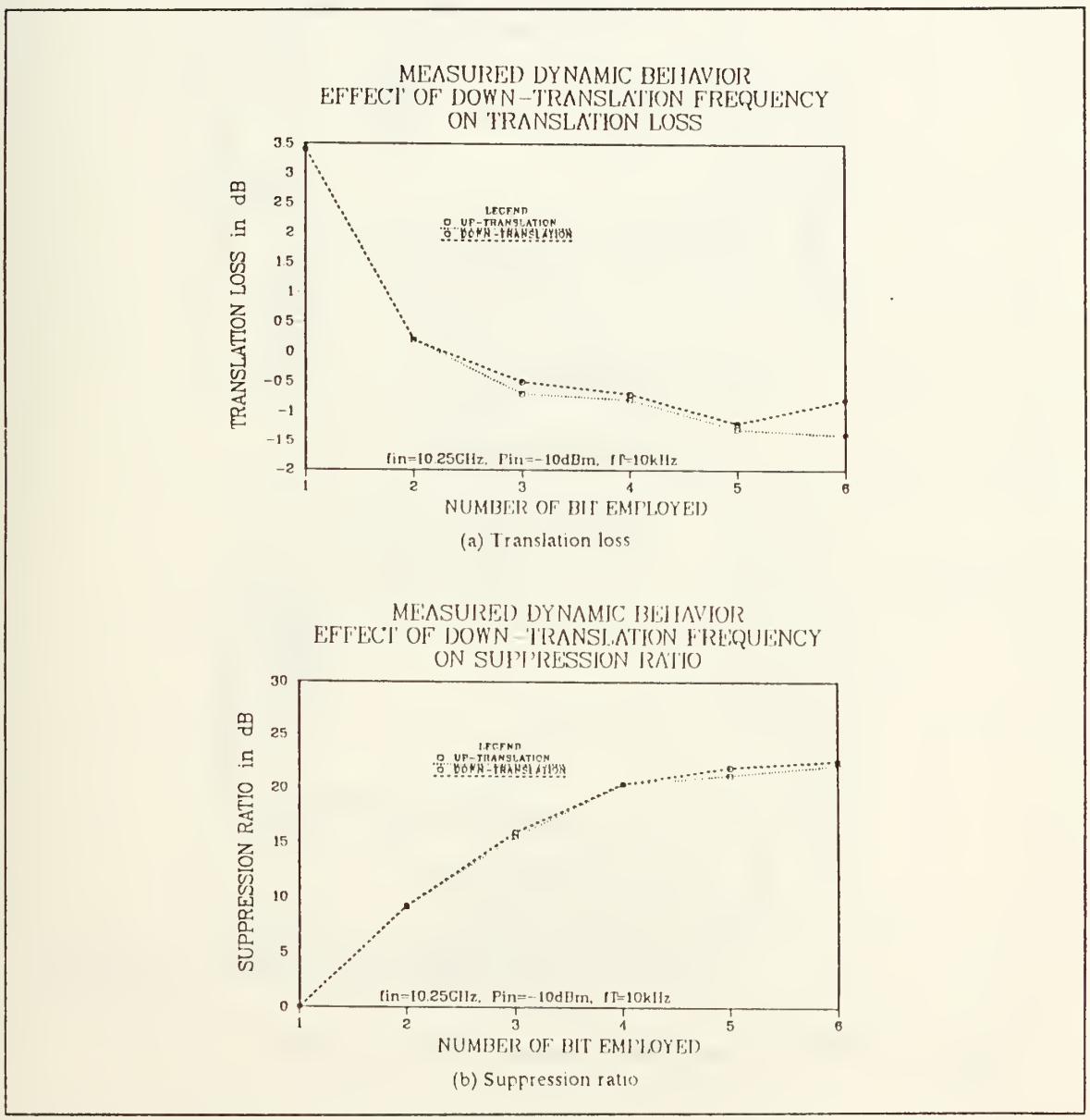


Figure 21. Effect of down-translation frequency on T.L. and S.R.

- Effect of translation frequency on T.L. and S.R. (See Figure 22)

  It can be seen that when the translation frequency is larger than 10 kHz, the suppression ratio will not be better than 25 dB. In order to have a suppression ratio of at least 20 dB, the translation frequency must be less than 20 kHz. At a larger value of translation frequency, only a small number of bits can be used due to the constraint of switching speed.

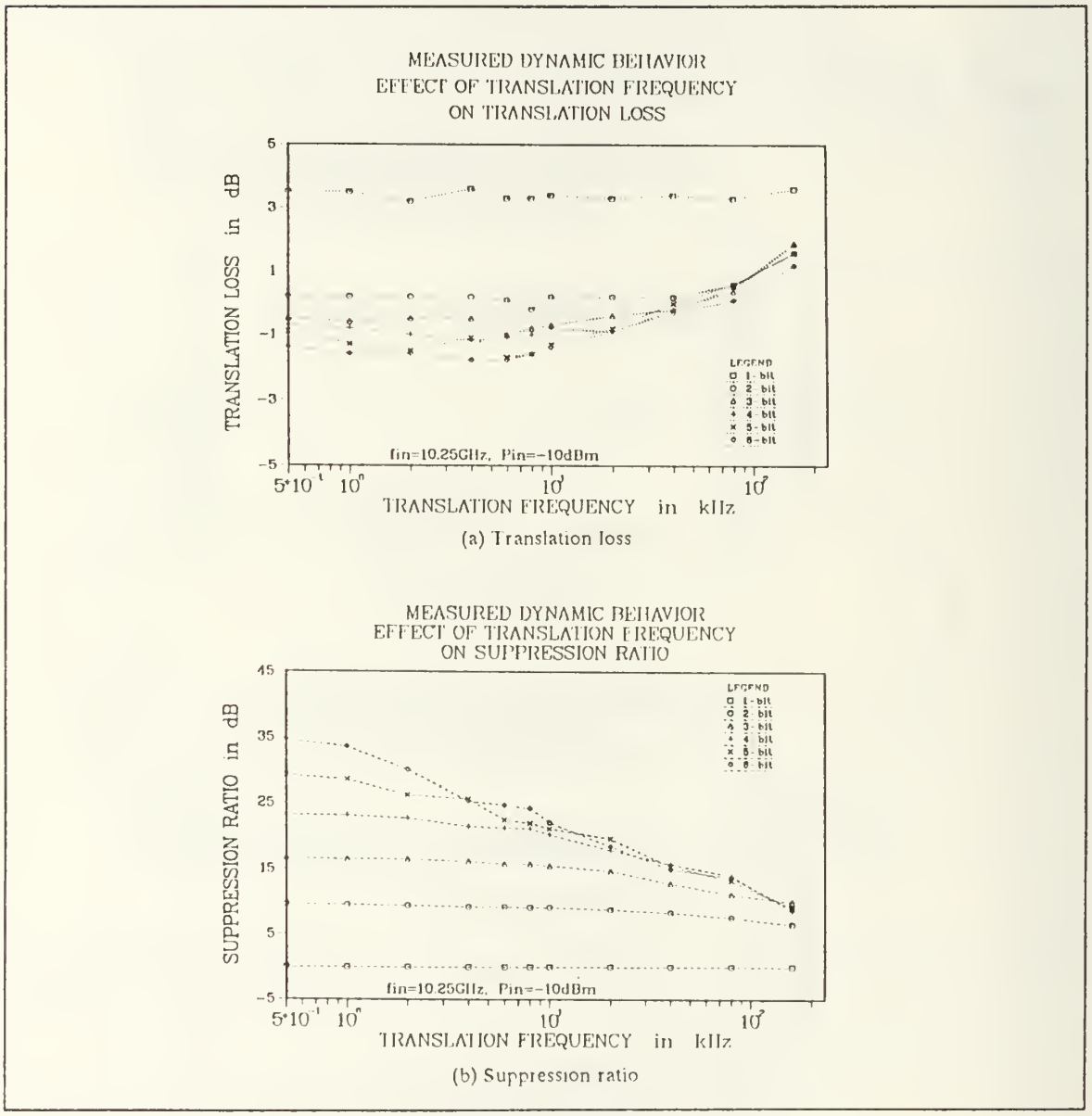


Figure 22. Effect of translation frequency on T.L. and S.R.

- Effect of carrier frequency on T.L. and S.R. (See Figure 23)

The carrier frequency has minimal effect on both the suppression ratio and the translation loss within the device operating frequency range.

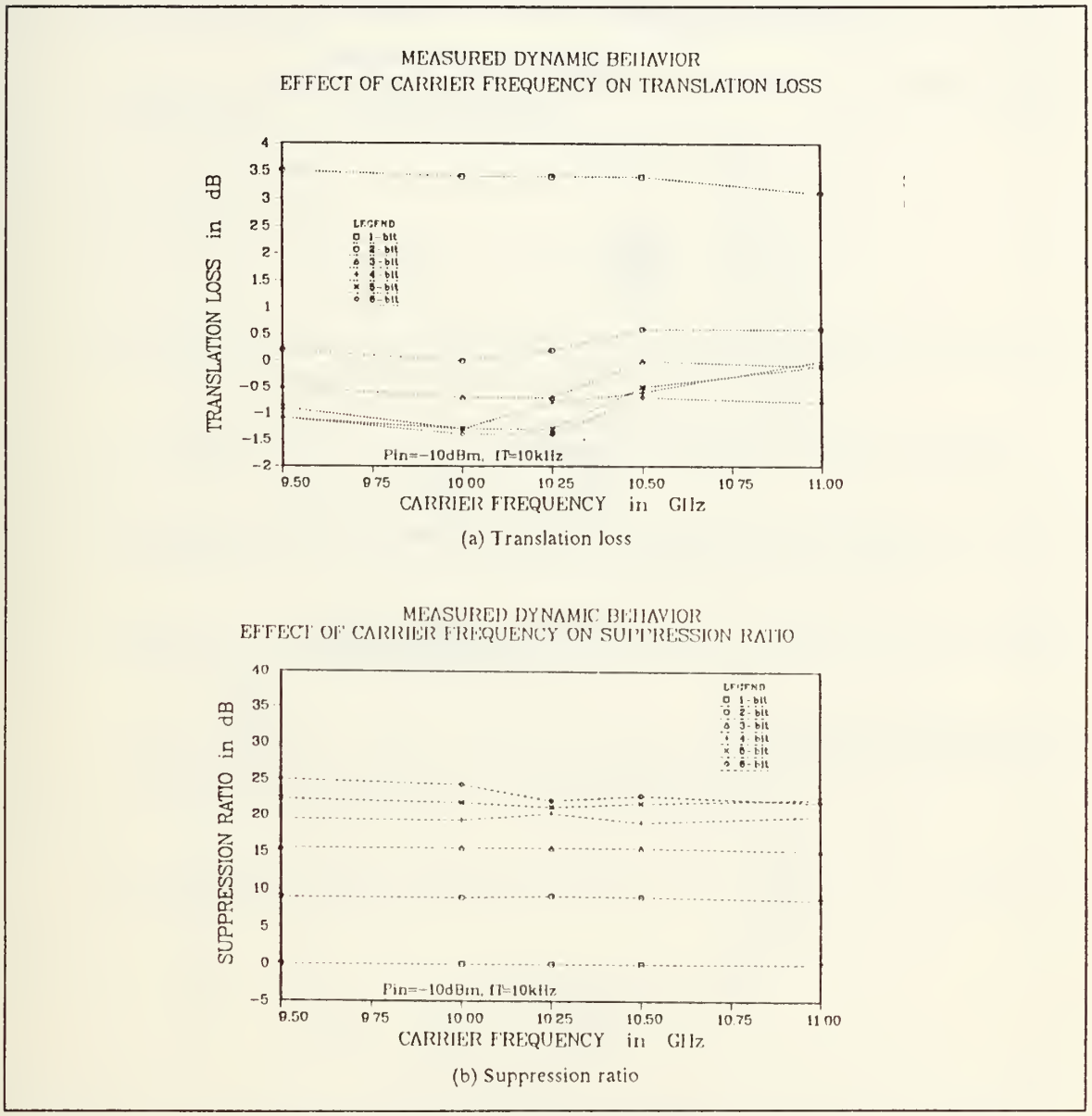


Figure 23. Effect of carrier frequency on T.L. and S.R.

- Effect of carrier input power on T.L. and S.R. (See Figure 24)

  The carrier input power has minimal effect on the suppression ratio and the translation loss.

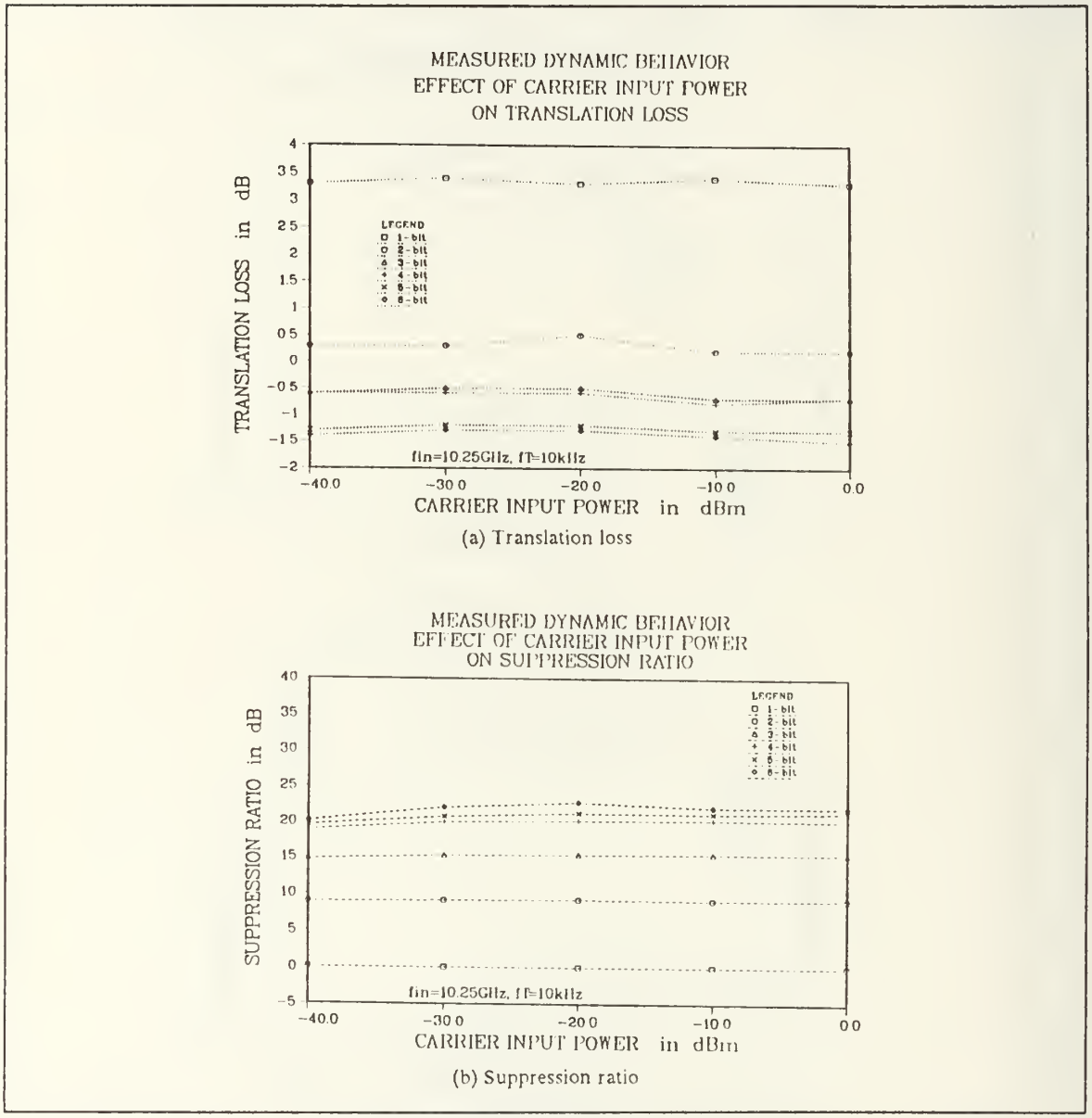


Figure 24. Effect of carrier input power on T.L. and S.R.

D. SUMMARY

- Static Characteristics
  - ▲ The phase shift characteristic of the component is fairly linear.
  - ▲ Both the insertion loss and the return loss have passed the manufacturer's specification.
- Serrodyning Effect
  - ▲ The spectral plots are similar to the theoretical ones.
  - ▲ The linewidth of the serrodyne waveform does not increase appreciably with respect to the linewidth of the incoming signal.
  - ▲ The noise floor is also raised substantially due to serrodyning operation.
  - ▲ The performance results follow the trends predicted theoretically and under simulation. However, the suppression capability is not as good.
  - ▲ Up- or down-translation will produce a signal of the corresponding frequency shift with essentially the same amount of T.L. and S.R..
  - ▲ The translation frequency affects the serrodyning performance the most. At higher translation frequencies, bits associated with small phase resolution do not contribute significantly to the suppression effect.
  - ▲ The carrier frequency and input power have been found to have minimal effect on the serrodyning performance.

VI. DISCUSSION

A. STUDY OF THE RESULTS GATHERED

1. Static Characteristics

▲ Phase Linearity

The phase shift characteristic of the component is fairly linear.

▲ Insertion Loss

Insertion loss (averaged S_{21}-dB value for the frequency of 10.25 GHz) is about 7.64 dB, which is better than the 8.5 dB specified by the manufacturer. Alternatively, when 0-bit is employed (zero translation frequency) in the study of the serrodyning effect, the monochromatic output (unserrodyned output) will also indicate the average insertion loss of the device. Generally, uneven insertion loss with respect to individual phase state in the static characteristics study (from Figure 18a) indicates the presence of an inherent undesired amplitude modulation effect during serrodyning operation.

▲ Reflection Loss

Since the maximum VSWR of this DPS is 1.8:1, the return loss must be at least 10.9 dB. The experimental reflection loss is found to be better than 12 dB.

2. Serrodyning Effect

▲ Spectral Plots in General

In serrodyning, the theoretical spectral plot consists of a series of zero frequency linewidth components with the main component located at $f_c + f_T$ Hz. The linewidth gets marginally broader with respect to the linewidth of the incoming signal in the simulated case and more so in the actual measurement.

▲ Effect of Number of Bits on Translation Loss and Suppression Ratio

The phase resolution depends on the number of bits available and employed. Figure 25a and Figure 25b are plots describing the comparison of theore-

tical, simulated and experimental T.L. and the S.R., respectively, against various number of bits employed.

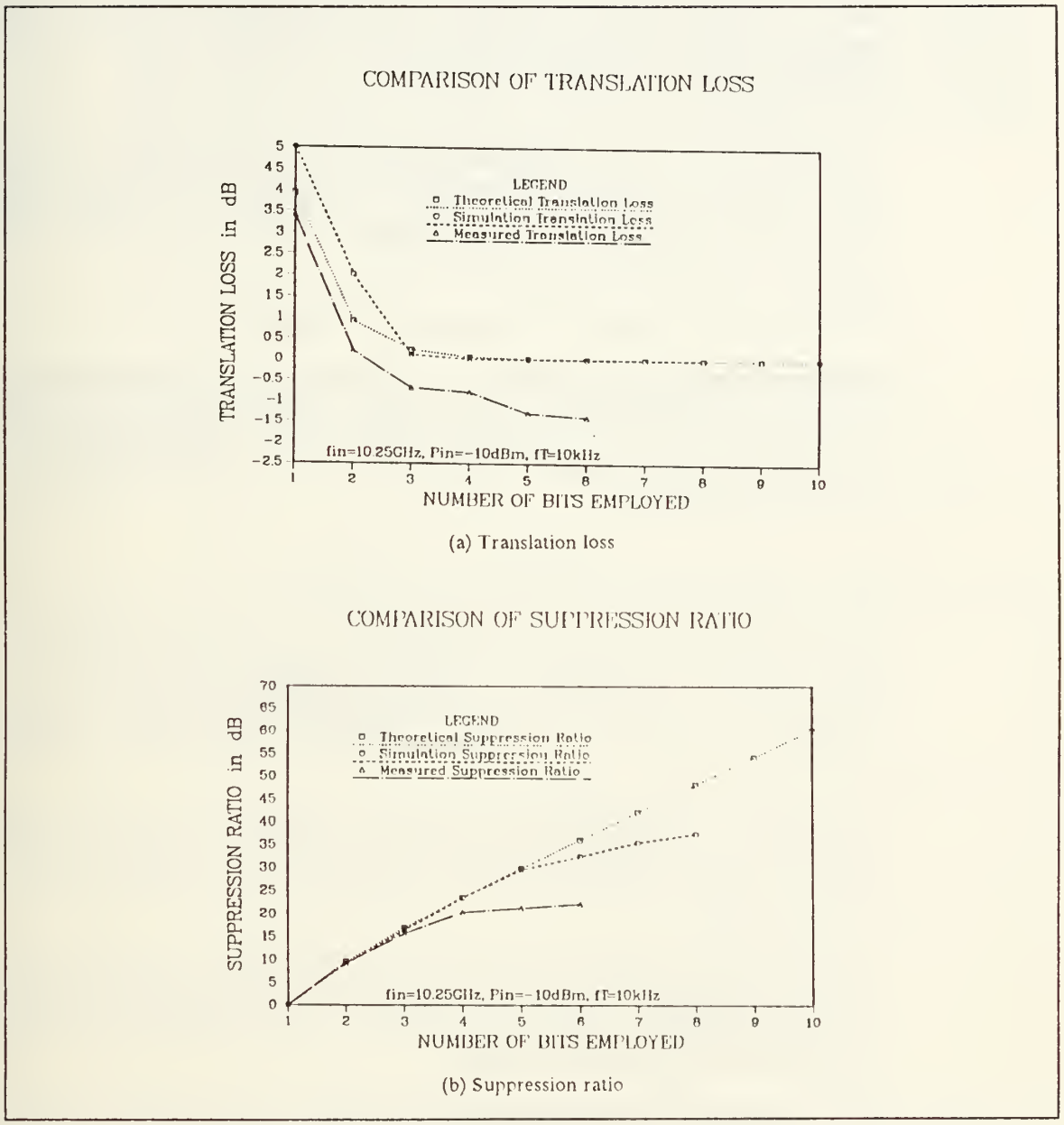


Figure 25. Comparison of translation loss and suppression ratio: Theoretical, simulation and experimental results.

Theoretical results show that the T.L. reduces very rapidly as the number of bits increases. At the same time, the S.R. improves. Both the T.L. and the S.R. exhibit the effect of diminishing marginal return as more bits are used.

The trends are followed by the results obtained under simulation. It is also noted that the S.R. is lower than the respective theoretical result. As for the T.L., it is higher at lower number of bits as compared with the theoretical result. Nonetheless, as the difference between the noise floor and the peak carrier power is only about 50 dB in our case. This difference cannot be distinguished for B greater than 9 bits.

The experimental results (characteristics obtained using the base set) do not correlate very well with the theoretical ones. In this case, the T.L. is lower than both the corresponding theoretical and simulated ones and even becomes negative for 3 or more bits. Moreover, the S.R. drops off more rapidly than in the theoretical and simulated cases.

- Effect of Down-Translation Frequency on T.L. and S.R.

The translation frequency is equal to the frequency of the 2π phase shift cycle. Up- or down-translation can be done by either up-shift or down-shift of the phase. Each case will produce a signal of the corresponding frequency shift with essentially the same amount of T.L. and S.R. (subject to the component tolerances).

- Effect of Translation Frequency on T.L. and S.R.

The switching speed of the device will dictate how large the translation frequency can be. For every serrodyning operation, translation frequency is constrained by the switching rate of the least significant phase bit (e.g., 5.6^{o} bit for $B = 6$ or 11.25^{o} bit for $B = 5$) which has to undergo a constant switching at a rate of $2^{B} \cdot f_{T}$ in order to approximate the staircase modulating waveform.

In this case, the switching speed of the device is specified to be 2 μs (switching frequency of 500 kHz). Therefore, for a translation utilizing 6 bits, the maximum theoretical translation frequency is only about 7.8 kHz ($\approx 500\text{kHz}/2^{6}$).

From the measured results (Figure 22a and Figure 22b), it can be seen that when the translation frequency is larger than 10 kHz, the S.R. will not be better than 25 dB. In order to have a S.R. of at least 20 dB, the translation frequency must be less than 20 kHz.

At a larger value of translation frequency, only a small number of bits can be used due to the constraint of switching speed. It should be noted that bits associated with small phase resolution do not contribute significantly to the suppression effect. Furthermore, external control circuit normally does not pose any problem on switching speed.

- Effect of Carrier Frequency on T.L. and S.R.

The carrier frequency has minimal effect on both the S.R. and the T.L. within the device operating frequency range. Normally, the phase accuracy

depends greatly on the range of the carrier frequency as well as the component design and the manufacturing process. For the transmission line type of digital phase shifter, the designed operating frequency range is normally narrow. Wide operating frequency requires a linear phase-shift characteristic, a built-in compensating network and a precise line length. Despite these additional requirements, the phase inaccuracy is still very large especially at the designed band edges. The RF vector modulation type of digital phase shifter, on the other hand, can usually be made to operate over a wider bandwidth with good phase accuracy.

- Effect of Carrier Input Power on T.L. and S.R.

  The carrier input power has minimal effect on the S.R. and the T.L..

B. STUDY OF VELOCITY GATE PULL-OFF AND RADAR RESPONSE

Figure 26 shows a typical scenario of a Velocity Gate Pull-Off (VGPO) [Refs. 19, 20 and 21].

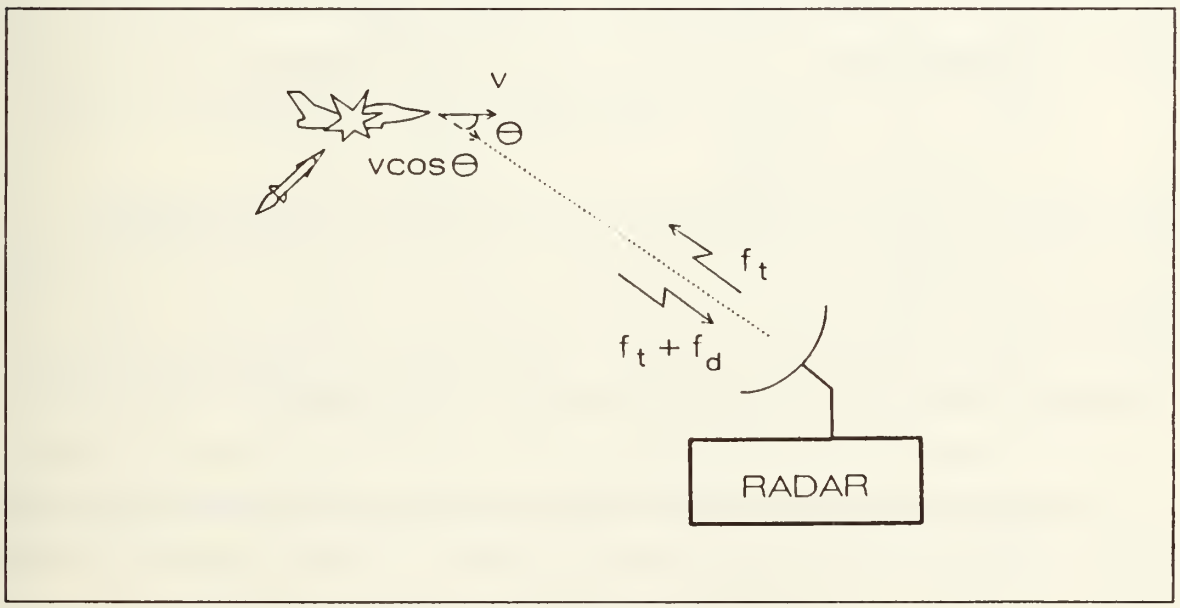


Figure 26. Velocity Gate Pull-Off (VGPO)

This technique is effective against a pulse or CW tracking radar which utilizes a doppler filter. The doppler velocity seen by the radar is given by Eq. (6-3). For f_d in kHz, f_t in GHz and V_R in knots, Eq. (6-4) can be obtained.

$$f_d = \frac{2V_R f_t}{C} \qquad (6-3)$$

$$f_d = \frac{V_R f_t}{291} \qquad (6-4)$$

For example, if f_t is 10 GHz and V_R is 500 knots, f_d at the radar is about 17.2 kHz. When the jammer is used, the doppler shift of the signal received by the jammer is about 8.6 kHz.

The signal is retransmitted by the jammer at the skin-echo frequency and this retransmitted signal is then slowly pulled away. Figure 27 shows a basic positively increasing false doppler return. Many other different serrodyning waveforms may also be employed depending on the scenarios.

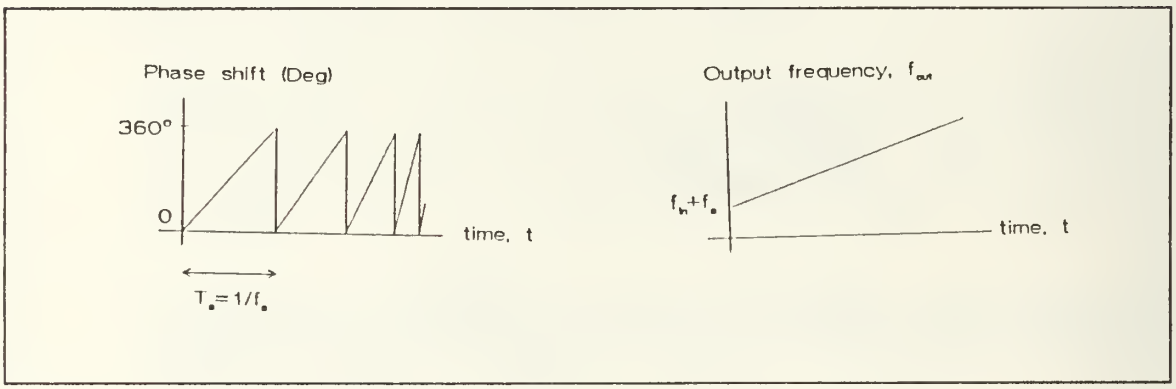


Figure 27. Example of a modulating waveform: From Ref. 19.

To ensure that the serrodyning is effective, the starting value of the doppler must first be within the speed gate of the radar in order for the radar to process this false signal. Secondly, the bandwidth of the shifted signal must be within the very narrow bandwidth of the radar's receiver velocity filter which is about 50 to 500 Hz. Thirdly, the retransmitted signal must remain within the radar filter for a sufficient duration so as to capture the radar's AGC and hence the velocity gate. The Jam-To-Signal Ratio Required (JSRR) is about 0 to 6 dB. Fourthly, the translation rate $\frac{df}{dt}$ must be slower than the radar tracking rate. When the radar tracks the false signal, the jammer will turn off the false signal sometime later so as to leave the radar with no signal and force the

radar to return to the acquisition mode again. Lastly, it is more effective to attempt to pull the radar doppler gate into the clutter region so that the tracker will lock on to the clutter. Figure 28 [Ref. 21] shows a typical deception program in which the solid line depicts a linear frequency translation and the dotted line an exponential (or parabolic) shift.

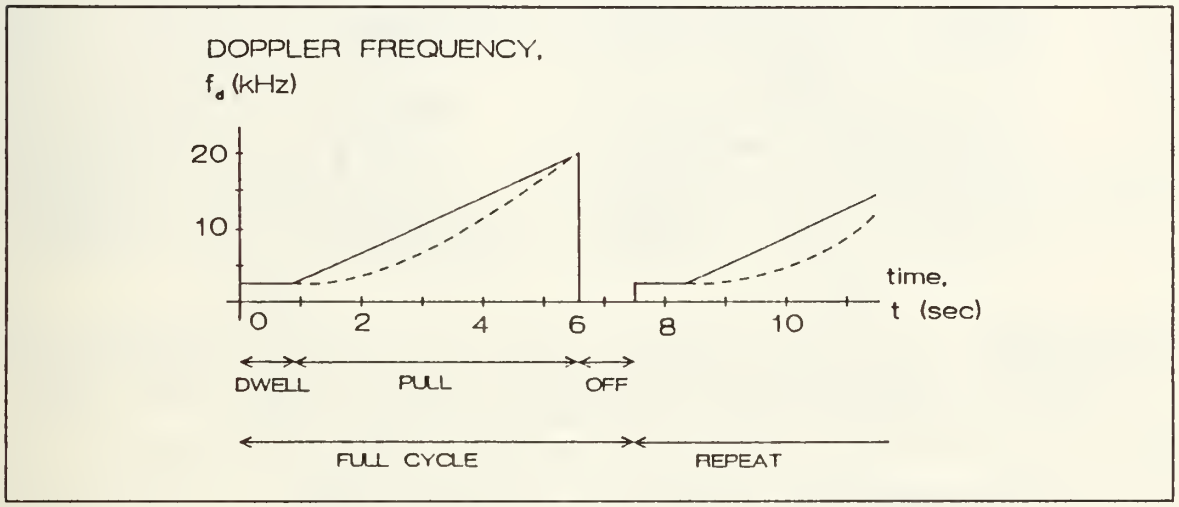


Figure 28. Example of a serrodyning program: From Ref. 21.

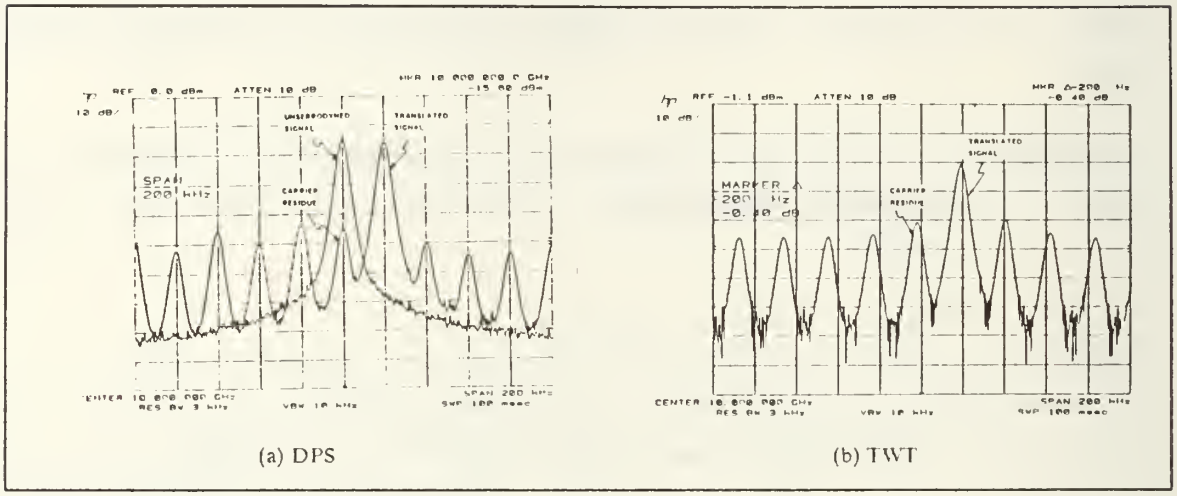
A doppler radar that is used for illumination or guidance is normally a single-target system. VGPO will be more effective than noise jamming since the bandwidth of the radar filter is very narrow. A doppler radar that can handle multiple targets normally employs a large number of doppler filters. In general, serrodyning is not very effective in this type of high-threat-density environment. To counter it, noise or a multiple-frequency repeater has to be used. Fast-tuned voltage control oscillators and digital microwave memory devices are also gaining more use in such an environment.

C. TRANSLATION LOSS AND SUPPRESSION RATIO

1. Carrier Suppression

Theoretically, in serrodyning employing DPS, the original carrier will be totally suppressed. Simulation also supports the argument. In the actual device measurement,

however, the residue of the carrier has been found to be present (See Figure 29a). The carrier residue has been suppressed by at least 25 dB when 6 bits are used.



(a) DPS (b) TWT

Figure 29. Spectral plot for DPS and TWT: With $f_c = 10$ GHz and $f_T = 20$ kHz. DPS uses 6 bits. The TWT used here is the HP494A.

2. Side Frequency Separation And Close-in Side Frequency

In serrodyning utilizing DPS, the undesired side frequency of the highest amplitude is located at a multiple (2^B) of translation frequency away from the main component. Figure 30 shows an example of the spectral plot using 6 bits, with the carrier frequency and translation frequency of 10 GHz and 10 kHz respectively.

The undesired side frequency of the highest amplitude is at 640 kHz away from the main component. It is, therefore, inferred that when a higher number of bits is being employed, the undesired component of greatest amplitude will be further away from the desired main component.

However, in practice, due to the narrow bandwidth of the radar's doppler filter, near side frequencies will be more undesirable in terms of degrading serrodyning effectiveness than those distant unwanted harmonic side frequencies, which may be of higher amplitude. Figure 29a and Figure 29b show the respective serrodyne waveform for DPS and TWT with near side frequencies vividly displayed. Clearly, the harmonic nearest the input signal frequency may be of greatest interest and one may prefer to de-

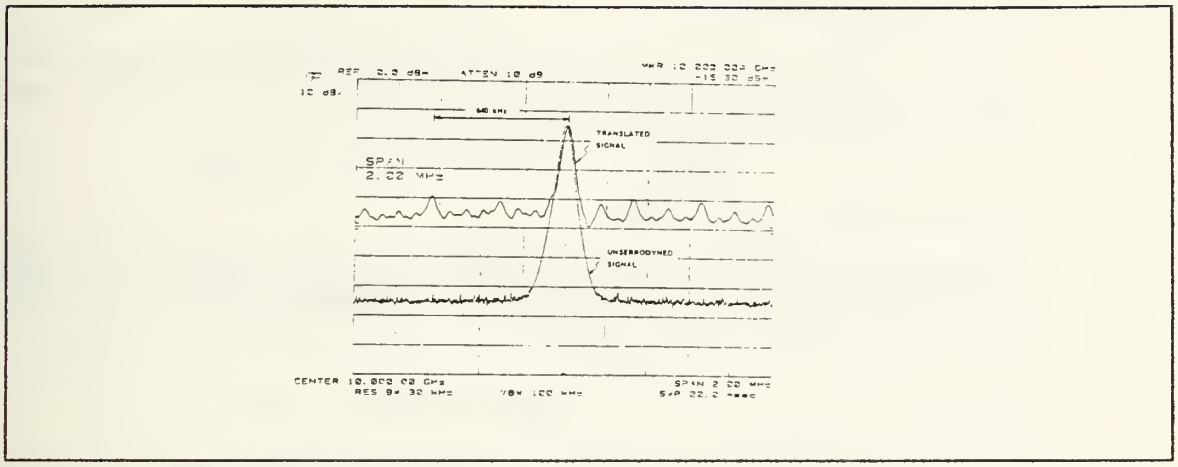


Figure 30. Side frequency separation for DPS: Carrier frequency = 10 GHz, translation frequency = 10 kHz and B = 6 bits.

fine S.R. with reference to this component. Therefore, practical S.R. might be defined relative to the nearest undesired side frequency rather than larger distant side frequencies. Following this definition, the suppression in this particular example is still more than 25 dB for the DPS in Figure 29a. This value is much better than 18.5 dB that follows the original theoretical definition (See Figure 22b).

D. COMPARISON OF TWT AND DIGITAL PHASE SHIFTER

Both the TWT and DPS can perform the serrodyning operation satisfactorily. The following is a comparison of the two methods based on experimental results.

- RF Frequency Band and Translation Frequency Range

  Both the TWT and DPS can be designed to have broad RF operating range. As for the translation frequency range, it is large for a TWT and generally small for a DPS. However, if the S.R. for the DPS is re-defined relative to the nearest undesired side frequency so as to give a more practical S.R. value, then the translation frequency range can also be very large. For example, if the switching speed of the device is specified to be 2 μs, the switching frequency is then 500 kHz. Therefore, for a frequency translation utilizing 6 bits, the maximum translation frequency is, theoretically, about 7.8 kHz. However, if only 5 bits are utilized, the maximum translation frequency can then be about 15.6 kHz. When only 1 bit is utilized, the maximum translation frequency is about 250 kHz.

- Gain and Insertion Loss

  Unlike a TWT, which can process the output signal with gain as well as frequency translation, the digital phase shifter has a large insertion loss due to the use of lossy PIN diodes and delay elements.

- Input and Output Power Level

  The carrier input power has a minimal effect on the S.R. and the T.L. for DPS. For a TWT, small signal gain is much higher than gain at saturation. A TWT also produces a higher power output.

- Saturation and PM-to-AM Conversion

  A TWT has a nonlinear power input-output characteristic and will saturate. Inherent undesired amplitude modulation is also present whenever the tube is phase modulated. The digital phase shifter does not have the saturation problem but its uneven insertion loss characteristic will introduce amplitude modulation.

- Translation Loss and Suppression Ratio

  The T.L. and the S.R. are very important parameters in the ECM application. When a TWT is used, the T.L. is not very critical since it provides gain by itself. Similarly, when using the digital phase shifter, even though its insertion loss may be large, the S.R. becomes a more significant parameter than the T.L. as gain provided by external amplification can compensate for the T.L..

- Noise and Signal Quality

  A TWT translator is low noise but side frequency harmonics occur at each multiple of the translation frequency. For example, if f_T is 10kHz, the undesired side frequencies for a TWT are at positive and negative multiple of f_T away from the main component. Whereas, for a DPS using 6 bits, the undesired side frequency of the highest amplitude is at 640 kHz away. Therefore, even if the S.R. are the same for the TWT and DPS, the DPS is obviously superior. However, if the close-in DPS side frequencies are deemed to be more undesirable in degrading serrodyning effectiveness, then the harmonics nearest to the input signal frequency will be dominant. In this case, the S.R. is still greater than 25 dB.

- Flyback Time

  With a TWT, a finite increase in flyback time will directly degrade the performance of serrodyning (i.e., increase in T.L. and reduction in S.R.). A digital phase shifter, strictly speaking, does not experience this problem of finite flyback time (See Figure 31). Rather, its performance will be limited by switching speed.

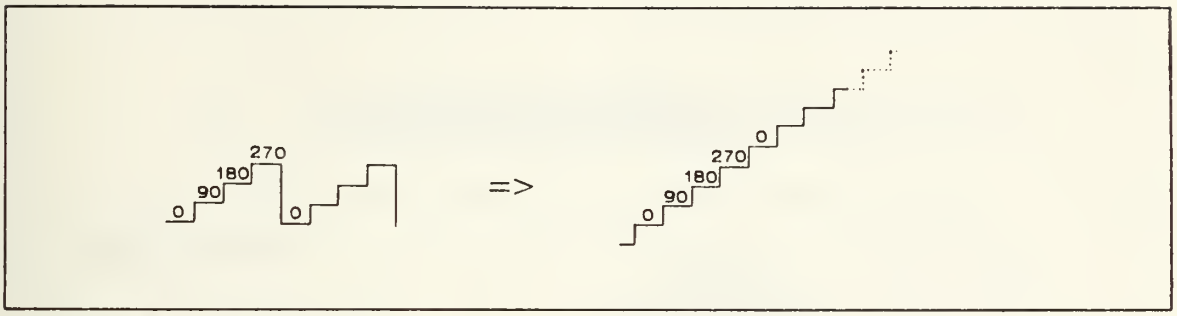


Figure 31. Flyback time

- Size and Ease of Implementation

  A DPS is compact, of solid-state design and relatively easy to interface with a computer and is programmable.

E. PRACTICAL USAGE

1. Criteria For Minimum Number Of Bits

The results gathered here can aid the design of a velocity deception ECM jammer. Since the measured results does not correlate very well with the theoretical and the simulated ones, the theoretical formulae can only be used as a first cut in the design procedure. The theoretical performance curve is shown in Figure 32.

For example, consider a specified serrodyning operation requiring a T.L. of not more than 0.5 dB and a S.R. of at least 25 dB. It can be seen that at least 3 bits are required for a T.L. of less than 0.5 dB and at least 5 bits are required for S.R. of more than 25 dB. Hence, the minimum number of bits required to achieve the desired serrodyne effect will then be 5. Taking into account other possible losses and minor imperfections in serrodyning operation, a 6-bit phase shifter is, therefore, recommended. Measurements should be made to check the performance.

For practical use, within the device limits, 6-bits can provide sufficient serrodyning capability. Whenever necessary, 1 bit should be added for safety reason.

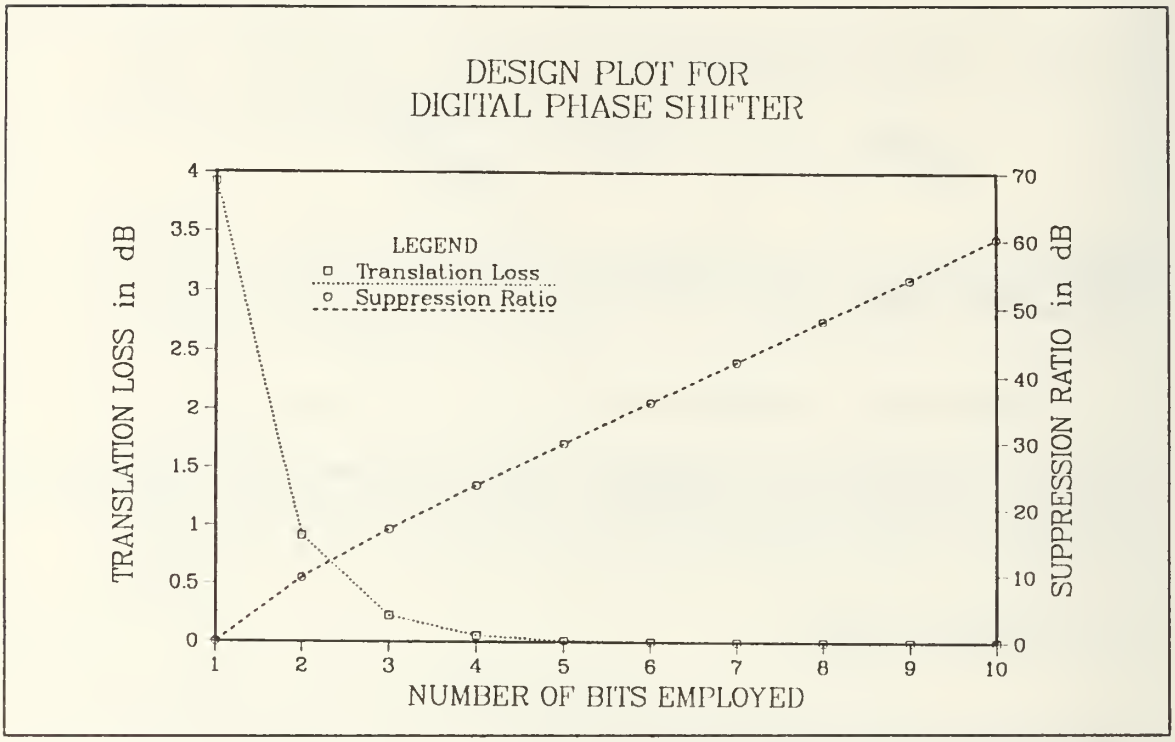


Figure 32. Design plot: With theoretical translation loss and suppression ratio superimposed.

2. Realistic Practical Model

Different models may be required for different types of DPS. In any case, a practical model must take into consideration the following :

- theoretical S.R. and T.L. limitations
- limitations due to switching speed of a digital phase shifter
- incidental amplitude modulation as a result of the uneven insertion loss
- non-linear phase characteristics
- near side frequency effect
- carrier suppression effect.

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Phase shifters are frequently used in ECM EW receiving as well as radar systems. This thesis has presented the theoretical analysis and the experimental validation of the serrodyning operation using digital phase shifters. The experimental results confirm the theory as well as the performance of the physical device. The results may be summarized as follows:

1. Theoretical and simulated results for the T.L. and the S.R. correlate rather well. As for the experimental results, the T.L. and the S.R. obtained, based on the theoretical definitions, are not very good.

2. Due to the narrow bandwidth of a radar's doppler filter, near side frequencies are less desirable in terms of degrading serrodyning effectiveness than those distant unwanted harmonic side frequencies, which may be of higher amplitude. Practical S.R. should be referenced to the nearest undesired side frequency rather than distant side frequencies. The S.R., in general, is more than 25 dB.

3. The original carrier is not totally suppressed; suppression of at least 25 dB were measured.

4. The linewidth of the serrodyne signal does not increase appreciably with respect to the linewidth of the incoming signal.

5. T.L. and S.R. for up- or down-translation are essentially the same.

6. The phase resolution depends on the number of bits available and employed.

7. Translation frequency affects the serrodyning performance most. The switching speed of the device will dictate how large the translation frequency can be. At higher translation frequencies, only a small number of bits can be used due to the constraint of switching speed. In addition, bits associated with small phase resolution do not contribute significantly to the suppression effect. Furthermore, external control circuitry normally does not limit switching speed.

8. The carrier frequency has been found to have minimal effect on both the T.L. and S.R. within the device operating frequency range.

9. The carrier input power has minimal effect on the T.L. and S.R..

10. To obtain good serrodyning performance, a system must have good S.R. and T.L., minimum PM-to-AM conversion and use high slew rate drivers.

11. A DPS is better than a TWT in implementing serrodyning for the following reasons:

    - A DPS is superior in terms of spectral purity.
    - A DPS does not experience the problem of finite flyback time.
    - A DPS is compact, of solid-state design, relatively easy to interface with a computer and is programmable.

B. RECOMMENDATIONS

A more realistic and practical model should be derived for each specific type of DPS. Otherwise, if theoretical formulae are used as design aids, measurements should be made to check the performance. For practical use, within the device limits, 6-bits can provide sufficient serrodyning capability. Whenever necessary, 1 bit should be added for safety reason.

APPENDIX A. DERIVATION

Similar results have also been derived by G. Klein and L. Dubrowsky [Ref. 5] using a different approach.

A. GENERAL EXPRESSION

Assuming a sinusoidal waveform $v_i(t)$ being phase-modulated by a step function $\theta(t)$. Then $v_s(t)$ is given by

$$v(t) = A\cos[\omega_c t + \theta(t)] \qquad (A-1.a)$$

$$= A\,Re\left[\, e^{j\omega_c t}\, e^{j\theta(t)}\right]. \qquad (A-1.b)$$

The phase $\theta(t)$ is periodic with frequency ω_m and $e^{j\theta(t)}$ can be represented by

$$e^{j\theta(t)} = \sum_{K=-\infty}^{\infty} C_K\, e^{jK\omega_m t}. \qquad (A-2)$$

Hence $v_s(t)$ can be rewritten as

$$v_s(t) = A \sum_{K=-\infty}^{\infty} C_K \cos(\omega_c + K\omega_m)t. \qquad (A-3)$$

where C_K is given by

$$C_K = \frac{1}{T_m} \int_{-\frac{T_m}{2}}^{\frac{T_m}{2}} e^{j\theta(t)}\, e^{-jK\omega_m t}\, dt \qquad (A-4.a)$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{j\theta(t)}\, e^{-j\frac{2\pi K}{T}t}\, dt. \qquad (A-4.b)$$

B. DERIVATION OF CONSTANT C_K

The phase $\theta(t)$ can be represented by a step function as shown in Figure 33. In this case, a positive rising ramp is assumed. The number N of steps is given by 2^B, where B

is the number of bits being employed in the serrodyning operation ($B \leq$ number of bits the DPS has).

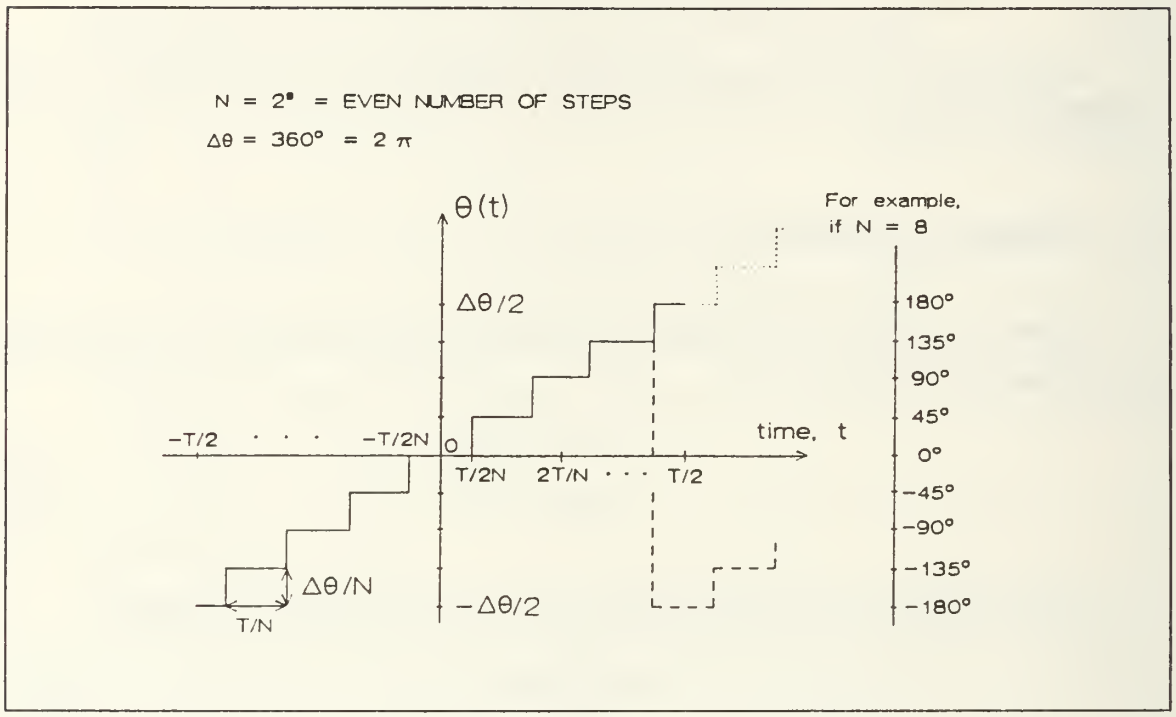


Figure 33. Modulation signal as a step function

The relationship of θ against t is summarized in Table 6. C_K becomes

$$C_K = \frac{1}{T}\left[\int_{\frac{T}{2}\left(1-\frac{1}{N}\right)}^{\frac{T}{2}} e^{j\frac{\Delta\theta}{2}} e^{-j\frac{2\pi K}{T}t}\,dt + \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}-1} \int_{\frac{T}{N}\left(n-\frac{1}{2}\right)}^{\frac{T}{N}\left(n+\frac{1}{2}\right)} e^{j\frac{\Delta\theta}{N}n} e^{-j\frac{2\pi K}{T}t}\,dt + \int_{-\frac{T}{2}}^{-\frac{T}{2}\left(1-\frac{1}{N}\right)} e^{-j\frac{\Delta\theta}{2}} e^{-j\frac{2\pi K}{T}t}\,dt\right] \qquad (A-5)$$

Table 6. PHASE ANGLE VERSUS TIME

Step number n	Range of t		Range of $\theta = \frac{\Delta\theta}{N} \cdot n$
	From	To	
$\frac{N}{2}$	$\frac{T}{2N}(N-1) = \frac{T}{2}\left(1-\frac{1}{N}\right)$	$\frac{T}{2}$	$\frac{\Delta\theta}{2}$
$\frac{N}{2}-1$	$\frac{T}{N}\left(n-\frac{1}{2}\right) = \frac{T}{2N}(N-3)$	$\frac{T}{N}\left(n+\frac{1}{2}\right) = \frac{T}{2N}(N-1)$	$\frac{\Delta\theta}{N}\left(\frac{N}{2}-1\right)$
⋮	⋮	⋮	⋮
2	$\frac{T}{N}\left(n-\frac{1}{2}\right) = \frac{3T}{2N}$	$\frac{T}{N}\left(n+\frac{1}{2}\right) = \frac{5T}{2N}$	$2\cdot\frac{\Delta\theta}{N}$
1	$\frac{T}{N}\left(n-\frac{1}{2}\right) = \frac{T}{2N}$	$\frac{T}{N}\left(n+\frac{1}{2}\right) = \frac{3T}{2N}$	$\frac{\Delta\theta}{N}$
0	$\frac{T}{N}\left(n-\frac{1}{2}\right) = -\frac{T}{2N}$	$\frac{T}{N}\left(n+\frac{1}{2}\right) = \frac{T}{2N}$	0
-1	$\frac{T}{N}\left(n-\frac{1}{2}\right) = -\frac{3}{2}\frac{T}{N}$	$\frac{T}{N}\left(n+\frac{1}{2}\right) = -\frac{T}{2N}$	$-\frac{\Delta\theta}{N}$
⋮	⋮	⋮	⋮
$-\left(\frac{N}{2}-1\right)$	$\frac{T}{N}\left(n-\frac{1}{2}\right) = -\frac{T}{2N}(N-1)$	$\frac{T}{N}\left(n+\frac{1}{2}\right) = -\frac{T}{2N}(N-3)$	$-\frac{\Delta\theta}{N}\left(\frac{N}{2}-1\right)$
$-\frac{N}{2}$	$-\frac{T}{2}$	$-\frac{T}{2N}(N-1) = \frac{-T}{2}\left(1-\frac{1}{N}\right)$	$-\frac{\Delta\theta}{2}$

Solving Eq. (A-5) gives

$$C_K = \frac{1}{T}\left[-\frac{1}{j\frac{2\pi K}{T}} e^{jn} e^{-j\frac{2\pi K}{T}t}\Bigg|_{\frac{T}{2}}^{\frac{T}{2}\left(1-\frac{1}{N}\right)} - \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}-1} \frac{1}{j\frac{2\pi K}{T}} e^{j\frac{\Delta\theta}{N}n} e^{-j\frac{2\pi K}{T}t}\Bigg|_{\frac{T}{N}\left(n-\frac{1}{2}\right)}^{\frac{T}{N}\left(n+\frac{1}{2}\right)} \right.$$

$$\left. -\frac{1}{j\frac{2\pi K}{T}} e^{-jn} e^{-j\frac{2\pi K}{T}t}\Bigg|_{-\frac{T}{2}}^{-\frac{T}{2}\left(1-\frac{1}{N}\right)} \right] \qquad (A-6.a)$$

$$= -\frac{1}{j2\pi K}\left[-e^{-j\frac{2\pi K}{T}t}\Big|_{\frac{T}{2}(1-\frac{1}{N})}^{\frac{T}{2}} + \sum_{n=-(\frac{N}{2}-1)}^{\frac{N}{2}-1} e^{j\frac{\Delta\theta}{N}n} e^{-j\frac{2\pi K}{T}t}\Big|_{\frac{T}{N}(n-\frac{1}{2})}^{\frac{T}{N}(n+\frac{1}{2})} \right. \tag{A-6.b}$$

$$\left. -e^{-j\frac{2\pi K}{T}t}\Big|_{-\frac{T}{2}}^{-\frac{T}{2}(1-\frac{1}{N})} \right].$$

Evaluating the first and the last terms in Eq. (A-6.b) gives

$$-\frac{1}{j2\pi K}\left[-e^{-j\frac{2\pi K}{T}t}\Big|_{\frac{T}{2}(1-\frac{1}{N})}^{\frac{T}{2}} - e^{-j\frac{2\pi K}{T}t}\Big|_{-\frac{T}{2}}^{-\frac{T}{2}(1-\frac{1}{N})} \right] \tag{A-7.a}$$

$$= -\frac{1}{j2\pi K}\left[-e^{-j\pi K} + e^{-j\pi K(1-\frac{1}{N})} - e^{j\pi K(1-\frac{1}{N})} + e^{j\pi K} \right] \tag{A-7.b}$$

$$= -\frac{1}{j2\pi K}\left\{ 2j\sin\pi K - 2j\sin\left[K\pi\left(1-\frac{1}{N}\right)\right] \right\} \tag{A-7.c}$$

$$= -\frac{\sin\pi K}{\pi K} + \frac{\sin\pi K\cos\frac{\pi K}{N}}{\pi K} - \frac{\cos\pi K\sin\frac{\pi K}{N}}{\pi K} \tag{A-7.d}$$

$$= \frac{\sin\pi K}{\pi K}\left(-1+\cos\frac{\pi K}{N}\right) - \frac{\cos\pi K\sin\frac{\pi K}{N}}{\pi K} \tag{A-7.e}$$

$$= -\frac{\cos K\pi\sin\frac{K\pi}{N}}{K\pi}. \tag{A-7.f}$$

Again, after considering Eq. (A-6.b) and evaluating the $\sum$ terms, it is found that there is a total of $N-1$ terms. Hence, the following can be obtained,

$$-\frac{1}{j2\pi K}\sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}-1} e^{j\frac{\Delta\theta}{N}n} e^{-j\frac{2\pi K}{T}t}\Big|_{\frac{T}{N}\left(n-\frac{1}{2}\right)}^{\frac{T}{N}\left(n+\frac{1}{2}\right)} \tag{A-8.a}$$

$$= -\frac{1}{j2\pi K}\sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}-1}\left\{ e^{j\left[\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(n+\frac{1}{2}\right)\right]} - e^{j\left[\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(n-\frac{1}{2}\right)\right]} \right\} \tag{A-8.b}$$

for the term $n = 0$,

$$-\frac{1}{j2\pi K}\left(e^{-j\frac{\pi K}{N}}-e^{j\frac{\pi K}{N}}\right)=\frac{\sin\frac{K\pi}{N}}{K\pi} \tag{A-9}$$

and for each pair of n and $-n$ terms, there is only a total of $\frac{N}{2}-1$ pairs,

$$-\frac{1}{j2\pi K}\sum_{n=1}^{\frac{N}{2}-1}\left\{e^{j\left[\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(n+\frac{1}{2}\right)\right]}-e^{j\left[\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(n-\frac{1}{2}\right)\right]}+e^{j\left[-\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(-n+\frac{1}{2}\right)\right]}-e^{j\left[-\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(-n-\frac{1}{2}\right)\right]}\right\} \tag{A-10.a}$$

$$=-\frac{1}{j2\pi K}\sum_{n=1}^{\frac{N}{2}-1}\left\{2j\sin\left[\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(n+\frac{1}{2}\right)\right]-2j\sin\left[\frac{\Delta\theta}{N}n-\frac{2\pi K}{N}\left(n-\frac{1}{2}\right)\right]\right\} \tag{A-10.b}$$

$$=\sum_{n=1}^{\frac{N}{2}-1}-\frac{1}{\pi K}\left\{\sin\left[\frac{2\pi n(1-K)}{N}-\frac{\pi K}{N}\right]-\sin\left[\frac{2\pi n(1-K)}{N}+\frac{\pi K}{N}\right]\right\} \tag{A-10.c}$$

$$=\sum_{n=1}^{\frac{N}{2}-1}-\frac{1}{\pi K}\left[-2\cos\frac{2\pi n(1-K)}{N}\sin\frac{\pi K}{N}\right] \tag{A-10.d}$$

$$=\sum_{n=1}^{\frac{N}{2}-1}2\frac{\sin\frac{K\pi}{N}}{K\pi}\cos\frac{2\pi n(1-K)}{N}. \tag{A-10.e}$$

Combining the terms, the overall $\sum$ terms can be written as

$$\frac{\sin\frac{K\pi}{N}}{K\pi}\left\{1+2\sum_{n=1}^{\frac{N}{2}-1}\cos\left[\frac{2\pi n}{N}(1-K)\right]\right\}. \tag{A-11}$$

C_K can then be deduced as

$$C_K=\frac{\sin\frac{K\pi}{N}}{K\pi}\left\{1-\cos K\pi+2\sum_{\substack{n=1\\N\neq 2}}^{\frac{N}{2}-1}\cos\left[\frac{2\pi n}{N}(1-K)\right]\right\}. \tag{A-12}$$

For $N = 2$ case, Eq. (A-12) becomes

$$C_K = \frac{\sin \frac{K\pi}{N}}{K\pi} \{ 1 - \cos K\pi \}. \qquad (A-13)$$

Let $K = mN + i$ for $i = 0,1$. Then Eq. (A-13) becomes

$$C_K = \frac{\sin \frac{K\pi}{N}}{K\pi} \{ 1 - \cos(mN + i)\pi \} \qquad (A-14.a)$$

$$= \frac{\sin \frac{K\pi}{N}}{K\pi} \{ 1 - \cos i\pi \}. \qquad (A-14.b)$$

From Eq. (A-14.b), it can be seen that C_K is zero for $i = 0$. As for $i = 1$, C_K is

$$C_K = 2 \cdot \frac{\sin \frac{K\pi}{N}}{K\pi}. \qquad (A-15)$$

For all $N \neq 2$ cases, using the equation for sum of cosine series [Ref. 22],

$$\cos \alpha + \cos 2\alpha + \cdots + \cos n\alpha = \frac{\sin\left(n + \frac{1}{2}\right)\alpha}{2 \sin \frac{\alpha}{2}} - \frac{1}{2} \qquad (A-16)$$

the $\sum$ terms in Eq. (A-12) can be simplified to

$$2 \sum_{\substack{n=1 \\ N \neq 2}}^{\frac{N}{2}-1} \cos n \left[\frac{2\pi}{N}(1 - K) \right] \qquad (A-17.a)$$

$$= \frac{\sin k\pi \cos \left[\frac{\pi}{N}(1 - K) \right]}{\sin \frac{\pi}{N}(1 - K)} + \cos K\pi - 1. \qquad (A-17.b)$$

Then Eq. (A-12) becomes

$$C_K = \frac{\sin \frac{K\pi}{N}}{K\pi} \left\{ \frac{\sin k\pi \cos \left[\frac{\pi}{N}(1 - K) \right]}{\sin \frac{\pi}{N}(1 - K)} \right\}. \qquad (A-18)$$

Again, for $K = mN + i$ for $i = 0,1,2, \ldots, N - 1$, Eq. (A-18) becomes

$$C_K = \frac{\sin \frac{K\pi}{N}}{K\pi} \left\{ \frac{\sin(mN + i)\pi \cos \left[\frac{\pi}{N}(1 - mN - i) \right]}{\sin \frac{\pi}{N}(1 - mN - i)} \right\} \qquad (A-19.a)$$

$$= \frac{\sin \frac{K\pi}{N}}{K\pi} \left\{ \frac{\sin mN\pi \cos l\pi \cos \left[\frac{\pi}{N}(1 - mN - l) \right]}{\sin \frac{\pi}{N}(1 - mN - l)} \right\}. \qquad (A-19.b)$$

For all $i \neq 1$ cases, Eq. (A-19.b) can be evaluated to be zero. As for $i = 1$ case, Eq. (A-19.b) becomes

$$C_K = - \frac{\sin \frac{K\pi}{N}}{K\pi} \left\{ \frac{\sin mN\pi \cos i\pi \cos m\pi}{\sin m\pi} \right\} \qquad (A-20.a)$$

$$= \frac{\sin \frac{K\pi}{N}}{K\pi} \left\{ \frac{\sin mN\pi}{\sin m\pi} (-1)^m \right\}. \qquad (A-20.b)$$

By L'Hôpital's Rule, Eq. (A-20.b) can be evaluated as

$$C_K = \frac{\sin \frac{K\pi}{N}}{K\pi} \left\{ \frac{N\pi}{\pi} \frac{\cos mN\pi}{\cos m\pi} (-1)^m \right\} \qquad (A-21.a)$$

$$= \frac{\sin \frac{K\pi}{N}}{K\pi} \cdot N. \qquad (A-21.b)$$

Therefore, for any $N = 2^B$ values, C_K is given by

$$C_K = \begin{cases} 0 & \text{for } K \neq m \cdot N + 1 \\ \dfrac{\sin \frac{K\pi}{N}}{K\pi} \cdot N & \text{for } K = m \cdot N + 1 \end{cases} \qquad (A-22)$$

The term $\dfrac{\sin \frac{K\pi}{N}}{K\pi}$ in Eq. (A-22) can be simplified as

$$\frac{\sin \frac{K\pi}{N}}{K\pi} = \frac{(-1)^m \sin \frac{\pi}{N}}{(mN + 1)\pi}. \qquad (A-23)$$

Hence the expression for C_K can be further simplified as

$$C_K = \frac{(-1)^m}{mN + 1} \frac{\sin \frac{\pi}{N}}{\frac{\pi}{N}}. \qquad (A-24)$$

Therefore, Eq. (A-3) can be re-written as

$$v_s(t) = A \sum_{\substack{K=-\infty \\ K=m\cdot N+1 \\ m=\text{integer}}}^{\infty} \frac{(-1)^m}{mN+1} \frac{\sin\frac{\pi}{N}}{\frac{\pi}{N}} \cdot \cos(\omega_c + K\omega_m)t. \qquad (A-25)$$

C. EXPRESSION FOR SPECTRAL COMPONENTS

For an unserrodyned signal, the average power P_0 is given by

$$P_0 = \frac{A^2}{2}. \qquad (A-26)$$

In the case of a serrodyne signal $v_s(t)$, the expression for average power P_K for correponding K terms is given by

$$P_K = C_K^2 \frac{A^2}{2}. \qquad (A-27)$$

The spectral plot is obtained with respect to unserrodyned signal and is

$$P_{K/0} = 10 \log \frac{P_K}{P_0} \qquad (A-28.a)$$

$$= 20 \log |C_K|. \qquad (A-28.b)$$

D. RELATIVE AMPLITUDES OF THE HARMONIC COMPONENTS

The fundamental component P_1 always has the largest magnitude. As for the harmonic components, it will be shown here that the first lower harmonic P_{-N+1} is larger than the first upper harmonic P_{N+1}. The ratio of P_{-N+1} over P_{N+1} is thus

$$\frac{P_{-N+1}}{P_{N+1}} = \left(\frac{C_{-N+1}}{C_{N+1}} \right)^2. \qquad (A-29)$$

C_{-N+1} and C_{N+1} become respectively,

$$C_{-N+1} = \frac{-1}{-N+1} \frac{\sin\frac{\pi}{N}}{\frac{\pi}{N}} \qquad (A-30)$$

$$C_{N+1} = \frac{-1}{N+1} \frac{\sin \frac{\pi}{N}}{\frac{\pi}{N}}. \qquad (A-31)$$

Substituting Eq. (A-30) and (A-31) for Eq. (A-29) gives

$$\frac{P_{-N+1}}{P_{N+1}} = \left(\frac{N+1}{N-1} \right)^2. \qquad (A-32)$$

The ratio given by Eq. (A-32) is always greater than 1 and approaches 1 as N gets larger.

E. THEORETICAL S.R. AND T.L.

The translation loss and the suppression ratio for different numbers of bits are tabulated in Table 7.

Table 7. THEORETICAL T.L. AND S.R.

Bit	T.L. (dB)	S.R. (dB)
1	3.92	0
2	0.912	9.54
3	0.224	16.9
4	0.0559	23.5
5	0.0140	29.8
6	0.00349	36.0
7	0.000872	42.1
8	0.000218	48.1
9	0.0000545	54.2
10	0.0000136	60.2

APPENDIX B. C_K FOR 1- TO 10-BIT

K	$C_K(1-B)$	$C_K(2-B)$	$C_K(3-B)$	$C_K(4-B)$	$C_K(5-B)$	$C_K(6-B)$	$C_K(7-B)$	$C_K(8-B)$	$C_K(9-B)$	$C_K(10-B)$
-199	-.31994E-02	-.45245E-02	0.48961E-02							
-198										
-197	0.32313E-02									
-196										
-195	-.32651E-02	0.46163E-02								
-194										
-193	0.32982E-02									
-192										
-191	-.33335E-02	-.47142E-02	-.51025E-02	-.52024E-02	-.52276E-02	0.52334E-02				
-190										
-189	0.33679E-02									
-188										
-187	-.34049E-02	0.48144E-02								
-186										
-185	0.34407E-02									
-184										
-183	-.34793E-02	-.49203E-02	0.53248E-02							
-182										
-181	0.35167E-02									
-180										
-179	-.35570E-02	0.50292E-02								
-178										
-177	0.35962E-02									
-176										
-175	-.36384E-02	-.51450E-02	-.55688E-02	0.56771E-02						
-174										
-173	0.36793E-02									
-172										
-171	-.37236E-02	0.52644E-02								
-170										
-169	0.37667E-02									
-168										
-167	-.38124E-02	-.53916E-02	0.58352E-02							
-166										
-165	0.38580E-02									
-164										
-163	-.39060E-02	0.55226E-02								
-162										
-161	0.39538E-02									
-160										
-159	-.40043E-02	-.56629E-02	-.61295E-02	-.62496E-02	0.62785E-02					
-158										
-157	0.40544E-02									
-156										
-155	-.41078E-02	0.58084E-02								
-154										
-153	0.41605E-02									
-152										
-151	-.42165E-02	-.59626E-02	0.64528E-02							
-150										
-149	0.42721E-02									
-148										
-147	-.43313E-02	0.61241E-02								
-146										
-145	0.43899E-02									
-144										
-143	-.44526E-02	-.62964E-02	-.68151E-02	0.69474E-02						
-142										
-141	0.45143E-02									
-140										
-139	-.45808E-02	0.64764E-02								
-138										
-137	0.46460E-02									
-136										
-135	-.47160E-02	-.66697E-02	0.72183E-02							
-134										
-133	0.47863E-02									
-132										
-131	-.48601E-02	0.68717E-02								
-130										
-129	0.49346E-02									
-128										
-127	-.50131E-02	-.70893E-02	-.76734E-02	-.78237E-02	-.78616E-02	-.78711E-02	0.78727E-02			
-126										
-125	0.50926E-02									
-124										
-123	-.51762E-02	0.73192E-02								
-122										
-121	0.52608E-02									
-120										

K	$C_K(1-B)$	$C_K(2-B)$	$C_K(3-B)$	$C_K(4-B)$	$C_K(5-B)$	$C_K(6-B)$	$C_K(7-B)$	$C_K(8-B)$	$C_K(9-B)$	$C_K(10-B)$
-119	-.53503E-02	-.75661E-02	0.81879E-02							
-118										
-117	0.54406E-02									
-116										
-115	-.55365E-02	0.78282E-02								
-114										
-113	0.56331E-02									
-112										
-111	-.57361E-02	-.81116E-02	-.87799E-02	0.89502E-02						
-110										
-109	0.58397E-02									
-108										
-107	-.59501E-02	0.84133E-02								
-106										
-105	0.60627E-02									
-104										
-103	-.61812E-02	-.87419E-02	0.94609E-02							
-102										
-101	0.63030E-02									
-100										
-99	-.64308E-02	0.90938E-02								
-98										
-97	0.65628E-02									
-96										
-95	-.67016E-02	-.94774E-02	-.10258E-01	-.10459E-01	0.10509E-01					
-94										
-93	0.68451E-02									
-92										
-91	-.69962E-02	0.98935E-02								
-90										
-89	0.71527E-02									
-88										
-87	-.73178E-02	-.10349E-01	0.11201E-01							
-86										
-85	0.74893E-02									
-84										
-83	-.76705E-02	0.10847E-01								
-82										
-81	0.78592E-02									
-80										
-79	-.80587E-02	-.11397E-01	-.12336E-01	0.12577E-01						
-78										
-77	0.82675E-02									
-76										
-75	-.84885E-02	0.12004E-01								
-74										
-73	0.87206E-02									
-72										
-71	-.89667E-02	-.12681E-01	0.13725E-01							
-70										
-69	0.92262E-02									
-68										
-67	-.95021E-02	0.13437E-01								
-66										
-65	0.97938E-02									
-64										
-63	-.10105E-01	-.14291E-01	-.15468E-01	-.15771E-01	-.15848E-01	0.15866E-01				
-62										
-61	0.10436E-01									
-60										
-59	-.10790E-01	0.15260E-01								
-58										
-57	0.11168E-01									
-56										
-55	-.11575E-01	-.16370E-01	0.17718E-01							
-54										
-53	0.12011E-01									
-52										
-51	-.12483E-01	0.17653E-01								
-50										
-49	0.12992E-01									
-48										
-47	-.13545E-01	-.19156E-01	-.20734E-01	0.21140E-01						
-46										
-45	0.14147E-01									
-44										
-43	-.14805E-01	0.20937E-01								
-42										
-41	0.15527E-01									
-40										

K	$C_K(1-B)$	$C_K(2-B)$	$C_K(3-B)$	$C_K(4-B)$	$C_K(5-B)$	$C_K(6-B)$	$C_K(7-B)$	$C_K(8-B)$	$C_K(9-B)$	$C_K(10-B)$
-39	-.16324E-01	-.23085E-01	0.24987E-01							
-38										
-37	0.17206E-01									
-36										
-35	-.18190E-01	0.25723E-01								
-34										
-33	0.19291E-01									
-32										
-31	-.20536E-01	-.29043E-01	-.31436E-01	-.32051E-01	0.32206E-01					
-30										
-29	0.21952E-01									
-28										
-27	-.23579E-01	0.33345E-01								
-26										
-25	0.25464E-01									
-24										
-23	-.27680E-01	-.39145E-01	0.42369E-01							
-22										
-21	0.30315E-01									
-20										
-19	-.33507E-01	0.47384E-01								
-18										
-17	0.37448E-01									
-16										
-15	-.42442E-01	-.60021E-01	-.64967E-01	0.66239E-01						
-14										
-13	0.48970E-01									
-12										
-11	-.57875E-01	0.81846E-01								
-10										
-9	0.70735E-01									
-8										
-7	-.90946E-01	-.12862E+00	0.13921E+00							
-6										
-5	0.12732E+00									
-4										
-3	-.21221E+00	0.30010E+00								
-2										
-1	0.63662E+00									
0										
1	0.63662E+00	0.90032E+00	0.97449E+00	0.99359E+00	0.99839E+00	0.99960E+00	0.99990E+00	0.99997E+00	0.99999E+00	0.10000E+01
2										
3	-.21221E+00									
4										
5	0.12732E+00	-.18006E+00								
6										
7	-.90946E-01									
8										
9	0.70735E-01	0.10003E+00	-.10828E+00							
10										
11	-.57875E-01									
12										
13	0.48970E-01	-.69255E-01								
14										
15	-.42442E-01									
16										
17	0.37448E-01	0.52959E-01	0.57323E-01	-.58447E-01						
18										
19	-.33507E-01									
20										
21	0.30315E-01	-.42872E-01								
22										
23	-.27680E-01									
24										
25	0.25464E-01	0.36012E-01	-.38980E-01							
26										
27	-.23579E-01									
28										
29	0.21952E-01	-.31046E-01								
30										
31	-.20536E-01									
32										
33	0.19291E-01	0.27282E-01	0.29530E-01	0.30108E-01	-.30254E-01					
34										
35	-.18189E-01									
36										
37	0.17206E-01	-.24333E-01								
38										
39	-.16324E-01									
40										

K	$C_K(1-B)$	$C_K(2-B)$	$C_K(3-B)$	$C_K(4-B)$	$C_K(5-B)$	$C_K(6-B)$	$C_K(7-B)$	$C_K(8-B)$	$C_K(9-B)$	$C_K(10-B)$
41	0.15527E-01	0.21958E-01	-.23768E-01							
42										
43	-.14805E-01									
44										
45	0.14147E-01	-.20007E-01								
46										
47	-.13545E-01									
48										
49	0.12992E-01	0.18373E-01	0.19887E-01	-.20278E-01						
50										
51	-.12483E-01									
52										
53	0.12011E-01	-.16987E-01								
54										
55	-.11575E-01									
56										
57	0.11168E-01	0.15795E-01	-.17096E-01							
58										
59	-.10791E-01									
60										
61	0.10436E-01	-.14759E-01								
62										
63	-.10105E-01									
64										
65	0.97939E-02	0.13851E-01	0.14992E-01	0.15286E-01	0.15360E-01	-.15379E-01				
66										
67	-.95020E-02									
68										
69	0.92261E-02	-.13048E-01								
70										
71	-.89668E-02									
72										
73	0.87206E-02	0.12333E-01	-.13349E-01							
74										
75	-.84885E-02									
76										
77	0.82675E-02	-.11693E-01								
78										
79	-.80587E-02									
80										
81	0.78592E-02	0.11115E-01	0.12030E-01	-.12267E-01						
82										
83	-.76705E-02									
84										
85	0.74892E-02	-.10592E-01								
86										
87	-.73178E-02									
88										
89	0.71527E-02	0.10115E-01	-.10950E-01							
90										
91	-.69962E-02									
92										
93	0.68451E-02	-.96809E-02								
94										
95	-.67015E-02									
96										
97	0.65628E-02	0.92812E-02	0.10046E-01	0.10243E-01	-.10293E-01					
98										
99	-.64307E-02									
100										
101	0.63029E-02	-.89143E-02								
102										
103	-.61812E-02									
104										
105	0.60627E-02	0.85735E-02	-.92819E-02							
106										
107	-.59501E-02									
108										
109	0.58402E-02	-.82607E-02								
110										
111	-.57362E-02									
112										
113	0.56330E-02	0.79668E-02	0.86232E-02	-.87935E-02						
114										
115	-.55366E-02									
116										
117	0.54405E-02	-.76957E-02								
118										
119	-.53504E-02									
120										

K	$C_K(1-B)$	$C_K(2-B)$	$C_K(3-B)$	$C_K(4-B)$	$C_K(5-B)$	$C_K(6-B)$	$C_K(7-B)$	$C_K(8-B)$	$C_K(9-B)$	$C_K(10-B)$
121	0.52608E-02	0.74402E-02	-.80542E-02							
122										
123	-.51763E-02									
124										
125	0.50925E-02	-.72030E-02								
126										
127	-.50131E-02									
128										
129	0.49345E-02	0.69785E-02	0.75535E-02	0.77014E-02	0.77387E-02	0.77480E-02	-.77517E-02			
130										
131	-.48601E-02									
132										
133	0.47862E-02	-.67702E-02								
134										
135	-.47161E-02									
136										
137	0.46466E-02	0.65710E-02	-.71138E-02							
138										
139	-.45808E-02									
140										
141	0.45143E-02	-.63860E-02								
142										
143	-.44526E-02									
144										
145	0.43898E-02	0.62087E-02	0.67203E-02	-.68528E-02						
146										
147	-.43314E-02									
148										
149	0.42720E-02	-.60429E-02								
150										
151	-.42166E-02									
152										
153	0.41604E-02	0.58842E-02	-.63696E-02							
154										
155	-.41078E-02									
156										
157	0.40544E-02	-.57346E-02								
158										
159	-.40044E-02									
160										
161	0.39537E-02	0.55916E-02	0.60522E-02	0.61708E-02	-.62018E-02					
162										
163	-.39060E-02									
164										
165	0.38579E-02	-.54573E-02								
166										
167	-.38124E-02									
168										
169	0.37667E-02	0.53269E-02	-.57668E-02							
170										
171	-.37232E-02									
172										
173	0.36792E-02	-.52048E-02								
174										
175	-.36384E-02									
176										
177	0.35961E-02	0.50862E-02	0.55054E-02	-.56138E-02						
178										
179	-.35571E-02									
180										
181	0.35166E-02	-.49744E-02								
182										
183	-.34794E-02									
184										
185	0.34406E-02	0.48661E-02	-.52675E-02							
186										
187	-.34049E-02									
188										
189	0.33679E-02	-.47637E-02								
190										
191	-.33335E-02									
192										
193	0.32981E-02	0.46644E-02	0.50488E-02	0.51477E-02	0.51726E-02	-.51793E-02				
194										
195	-.32651E-02									
196										
197	0.32312E-02	-.45708E-02								
198										
199	-.31994E-02									
200										

APPENDIX C. P_K FOR 1- TO 10-BIT

K	$P_K(1-B)$	$P_K(2-B)$	$P_K(3-B)$	$P_K(4-B)$	$P_K(5-B)$	$P_K(6-B)$	$P_K(7-B)$	$P_K(8-B)$	$P_K(9-B)$	$P_K(10-B)$
-199	-.49899E+02	-.46899E+02	-.46203E+02	-.12944E+03	-.12977E+03	-.12998E+03	-.11909E+03	-.11605E+03	-.11359E+03	-.11956E+03
-198	-.12476E+03	-.12476E+03	-.12812E+03	-.12799E+03	-.11979E+03	-.15012E+03	-.10901E+03	-.14170E+03	-.12139E+03	-.12518E+03
-197	-.49813E+02	-.12133E+03	-.12685E+03	-.14460E+03	-.12371E+03	-.12524E+03	-.11390E+03	-.13147E+03	-.11366E+03	-.12293E+03
-196	-.20000E+03	-.12383E+03	-.13524E+03	-.15145E+03	-.12779E+03	-.13116E+03	-.13029E+03	-.13062E+03	-.12857E+03	-.13203E+03
-195	-.49722E+02	-.46714E+02	-.13667E+03	-.13493E+03	-.12575E+03	-.12214E+03	-.11965E+03	-.11377E+03	-.11110E+03	-.11709E+03
-194	-.12299E+03	-.17357E+03	-.12433E+03	-.12586E+03	-.13522E+03	-.12766E+03	-.11999E+03	-.11137E+03	-.12614E+03	-.12538E+03
-193	-.49635E+02	-.13548E+03	-.12629E+03	-.12689E+03	-.13674E+03	-.11989E+03	-.11525E+03	-.11184E+03	-.12490E+03	-.16542E+03
-192	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12167E+03	-.12407E+03	-.12192E+03	-.13701E+03	-.12658E+03
-191	-.49542E+02	-.46532E+02	-.45844E+02	-.45676E+02	-.45634E+02	-.45624E+02	-.12539E+03	-.11888E+03	-.11685E+03	-.12304E+03
-190	-.12149E+03	-.12150E+03	-.12003E+03	-.12002E+03	-.11902E+03	-.13495E+03	-.12839E+03	-.11996E+03	-.10803E+03	-.11879E+03
-189	-.49453E+02	-.12802E+03	-.12155E+03	-.11967E+03	-.12496E+03	-.13014E+03	-.12637E+03	-.16019E+03	-.11756E+03	-.13023E+03
-188	-.20000E+03	-.12080E+03	-.12878E+03	-.12366E+03	-.12271E+03	-.12324E+03	-.11531E+03	-.13513E+03	-.12811E+03	-.12307E+03
-187	-.49358E+02	-.46249E+02	-.12585E+03	-.13954E+03	-.12451E+03	-.13173E+03	-.11935E+03	-.12427E+03	-.12628E+03	-.12067E+03
-186	-.12015E+03	-.17868E+03	-.12252E+03	-.12136E+03	-.12154E+03	-.12298E+03	-.12600E+03	-.11854E+03	-.10201E+03	-.10878E+03
-185	-.49267E+02	-.14106E+03	-.12047E+03	-.11832E+03	-.12327E+03	-.13371E+03	-.11627E+03	-.13175E+03	-.11479E+03	-.13205E+03
-184	-.20000E+03	-.20000E+03	-.12397E+03	-.11751E+03	-.11803E+03	-.12657E+03	-.11210E+03	-.12522E+03	-.11689E+03	-.13004E+03
-183	-.49170E+02	-.46160E+02	-.45474E+02	-.12947E+03	-.12699E+03	-.13035E+03	-.11995E+03	-.12150E+03	-.11889E+03	-.12947E+03
-182	-.11895E+03	-.11896E+03	-.14558E+03	-.13520E+03	-.13814E+03	-.12595E+03	-.12481E+03	-.12118E+03	-.12104E+03	-.11825E+03
-181	-.49077E+02	-.12403E+03	-.13107E+03	-.12606E+03	-.12500E+03	-.12044E+03	-.11876E+03	-.10492E+03	-.11759E+03	-.11666E+03
-180	-.20000E+03	-.11838E+03	-.13336E+03	-.12111E+03	-.14718E+03	-.12469E+03	-.12465E+03	-.11979E+03	-.13409E+03	-.11959E+03
-179	-.48978E+02	-.45970E+02	-.13046E+03	-.12268E+03	-.14443E+03	-.12116E+03	-.12935E+03	-.13335E+03	-.10394E+03	-.13119E+03
-178	-.11784E+03	-.14558E+03	-.13175E+03	-.11892E+03	-.11730E+03	-.13334E+03	-.13019E+03	-.13711E+03	-.12491E+03	-.12858E+03
-177	-.48983E+02	-.13889E+03	-.12689E+03	-.13771E+03	-.12927E+03	-.14234E+03	-.12735E+03	-.12072E+03	-.13701E+03	-.14416E+03
-176	-.13312E+03	-.20000E+03	-.20000E+03	-.12346E+03	-.12426E+03	-.12624E+03	-.12316E+03	-.11503E+03	-.12452E+03	-.12640E+03
-175	-.48782E+02	-.45772E+02	-.45085E+02	-.44917E+02	-.12876E+03	-.12600E+03	-.11550E+03	-.11799E+03	-.12755E+03	-.12237E+03
-174	-.11682E+03	-.12057E+03	-.11848E+03	-.12806E+03	-.13253E+03	-.12710E+03	-.11521E+03	-.11694E+03	-.12332E+03	-.12158E+03
-173	-.48695E+02	-.12209E+03	-.11896E+03	-.13284E+03	-.12314E+03	-.12997E+03	-.14736E+03	-.11686E+03	-.11601E+03	-.12007E+03
-172	-.13292E+03	-.12589E+03	-.12357E+03	-.12167E+03	-.12536E+03	-.12190E+03	-.11511E+03	-.12768E+03	-.13917E+03	-.12525E+03
-171	-.48581E+02	-.45573E+02	-.12106E+03	-.13954E+03	-.11959E+03	-.11609E+03	-.13287E+03	-.12362E+03	-.11044E+03	-.99693E+02
-170	-.11587E+03	-.12590E+03	-.12744E+03	-.12946E+03	-.12489E+03	-.11659E+03	-.11362E+03	-.12446E+03	-.12469E+03	-.11073E+03
-169	-.48481E+02	-.12598E+03	-.12660E+03	-.12191E+03	-.11951E+03	-.12577E+03	-.12576E+03	-.10597E+03	-.11770E+03	-.11816E+03
-168	-.13272E+03	-.20000E+03	-.11860E+03	-.13448E+03	-.13034E+03	-.12971E+03	-.12756E+03	-.11044E+03	-.10633E+03	-.13522E+03
-167	-.48376E+02	-.45366E+02	-.44679E+02	-.12653E+03	-.13107E+03	-.12591E+03	-.14558E+03	-.13115E+03	-.11165E+03	-.11322E+03
-166	-.12143E+03	-.11801E+03	-.12271E+03	-.12753E+03	-.13228E+03	-.12216E+03	-.11654E+03	-.12493E+03	-.11957E+03	-.12193E+03
-165	-.48273E+02	-.11950E+03	-.12718E+03	-.14127E+03	-.14626E+03	-.11714E+03	-.12685E+03	-.11156E+03	-.13142E+03	-.11937E+03
-164	-.13251E+03	-.12184E+03	-.12544E+03	-.13084E+03	-.13559E+03	-.11000E+03	-.11582E+03	-.12541E+03	-.12191E+03	-.12777E+03
-163	-.48165E+02	-.45157E+02	-.13371E+03	-.14967E+03	-.12614E+03	-.12207E+03	-.12445E+03	-.12105E+03	-.11518E+03	-.11896E+03
-162	-.12228E+03	-.12536E+03	-.13217E+03	-.12796E+03	-.12097E+03	-.12431E+03	-.13370E+03	-.12182E+03	-.13266E+03	-.13748E+03
-161	-.48060E+02	-.13366E+03	-.12497E+03	-.12593E+03	-.12106E+03	-.12010E+03	-.12335E+03	-.12272E+03	-.11531E+03	-.12474E+03
-160	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.11842E+03	-.13397E+03	-.14329E+03	-.11838E+03	-.12101E+03	-.13044E+03
-159	-.47949E+02	-.44939E+02	-.44252E+02	-.44083E+02	-.44043E+02	-.12913E+03	-.11454E+03	-.12657E+03	-.11371E+03	-.11295E+03
-158	-.12061E+03	-.12122E+03	-.12281E+03	-.12045E+03	-.12215E+03	-.11612E+03	-.12962E+03	-.12871E+03	-.12276E+03	-.12031E+03
-157	-.47841E+02	-.12576E+03	-.12107E+03	-.11898E+03	-.12876E+03	-.12032E+03	-.11884E+03	-.11816E+03	-.12773E+03	-.12232E+03
-156	-.20000E+03	-.11995E+03	-.12281E+03	-.11949E+03	-.13859E+03	-.11410E+03	-.11997E+03	-.11937E+03	-.11422E+03	-.13011E+03
-155	-.47728E+02	-.44719E+02	-.12124E+03	-.14233E+03	-.14090E+03	-.13045E+03	-.11741E+03	-.11222E+03	-.12486E+03	-.12196E+03
-154	-.11914E+03	-.13436E+03	-.12164E+03	-.12753E+03	-.13123E+03	-.12526E+03	-.12764E+03	-.11277E+03	-.11724E+03	-.11362E+03
-153	-.47617E+02	-.14049E+03	-.13121E+03	-.11951E+03	-.13130E+03	-.11992E+03	-.12503E+03	-.10320E+03	-.12873E+03	-.11754E+03
-152	-.20000E+03	-.20000E+03	-.12183E+03	-.12024E+03	-.13513E+03	-.12658E+03	-.14736E+03	-.13663E+03	-.11489E+03	-.12399E+03
-151	-.47501E+02	-.44491E+02	-.43805E+02	-.12651E+03	-.11776E+03	-.12610E+03	-.13277E+03	-.13870E+03	-.11827E+03	-.12456E+03
-150	-.11979E+03	-.12096E+03	-.12576E+03	-.14106E+03	-.11726E+03	-.11593E+03	-.13417E+03	-.11294E+03	-.14289E+03	-.13281E+03
-149	-.47387E+02	-.12321E+03	-.13146E+03	-.14272E+03	-.12793E+03	-.12894E+03	-.11650E+03	-.12460E+03	-.12705E+03	-.12621E+03
-148	-.20000E+03	-.11721E+03	-.18664E+03	-.11731E+03	-.13905E+03	-.12347E+03	-.11766E+03	-.12205E+03	-.11545E+03	-.10705E+03
-147	-.47268E+02	-.44259E+02	-.12796E+03	-.12635E+03	-.11985E+03	-.11354E+03	-.13653E+03	-.10839E+03	-.97739E+02	-.12044E+03
-146	-.11835E+03	-.12794E+03	-.12533E+03	-.13214E+03	-.12535E+03	-.12416E+03	-.15301E+03	-.15175E+03	-.13153E+03	-.11947E+03
-145	-.47151E+02	-.13674E+03	-.12850E+03	-.12901E+03	-.12135E+03	-.15754E+03	-.11540E+03	-.15551E+03	-.14170E+03	-.12132E+03
-144	-.20000E+03	-.20000E+03	-.20000E+03	-.12029E+03	-.12765E+03	-.14082E+03	-.13133E+03	-.15429E+03	-.12468E+03	-.14213E+03
-143	-.47028E+02	-.44018E+02	-.43331E+02	-.43167E+02	-.12708E+03	-.12683E+03	-.14152E+03	-.11517E+03	-.12227E+03	-.13480E+03
-142	-.11706E+03	-.12289E+03	-.11989E+03	-.14239E+03	-.14130E+03	-.12136E+03	-.12352E+03	-.12941E+03	-.11420E+03	-.11183E+03
-141	-.46908E+02	-.11996E+03	-.11806E+03	-.12489E+03	-.11993E+03	-.12891E+03	-.12205E+03	-.10896E+03	-.11964E+03	-.12984E+03
-140	-.20000E+03	-.12346E+03	-.11702E+03	-.12965E+03	-.12439E+03	-.13133E+03	-.12635E+03	-.11349E+03	-.11871E+03	-.10899E+03
-139	-.46781E+02	-.43773E+02	-.12799E+03	-.12361E+03	-.11920E+03	-.12413E+03	-.12221E+03	-.12439E+03	-.15283E+03	-.12695E+03
-138	-.11587E+03	-.13690E+03	-.13359E+03	-.12014E+03	-.13158E+03	-.13524E+03	-.11416E+03	-.12817E+03	-.12831E+03	-.15868E+03
-137	-.46658E+02	-.13397E+03	-.12299E+03	-.11945E+03	-.12140E+03	-.12090E+03	-.11732E+03	-.11617E+03	-.10442E+03	-.12565E+03
-136	-.20000E+03	-.20000E+03	-.11607E+03	-.13059E+03	-.12708E+03	-.12561E+03	-.12216E+03	-.12678E+03	-.12655E+03	-.11958E+03
-135	-.46529E+02	-.43518E+02	-.42831E+02	-.12405E+03	-.12240E+03	-.12488E+03	-.12715E+03	-.11261E+03	-.11819E+03	-.11751E+03
-134	-.12354E+03	-.12185E+03	-.13729E+03	-.11962E+03	-.12114E+03	-.12446E+03	-.12691E+03	-.11491E+03	-.11581E+03	-.12195E+03
-133	-.46400E+02	-.11736E+03	-.13114E+03	-.13693E+03	-.12319E+03	-.12330E+03	-.12635E+03	-.13328E+03	-.13581E+03	-.12198E+03
-132	-.20000E+03	-.11953E+03	-.12278E+03	-.14163E+03	-.12089E+03	-.12591E+03	-.12034E+03	-.13905E+03	-.12087E+03	-.12163E+03
-131	-.46267E+02	-.43259E+02	-.13056E+03	-.14248E+03	-.12927E+03	-.13049E+03	-.15927E+03	-.12279E+03	-.11818E+03	-.14095E+03
-130	-.12131E+03	-.13644E+03	-.12050E+03	-.12246E+03	-.12935E+03	-.12527E+03	-.15711E+03	-.13447E+03	-.13014E+03	-.13145E+03
-129	-.46135E+02	-.14097E+03	-.12587E+03	-.12601E+03	-.13041E+03	-.13190E+03	-.12323E+03	-.11954E+03	-.11770E+03	-.12455E+03
-128	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.11595E+03	-.11937E+03	-.10556E+03	-.10813E+03
-127	-.45998E+02	-.42988E+02	-.42300E+02	-.42132E+02	-.42090E+02	-.42079E+02	-.42078E+02	-.13089E+03	-.13415E+03	-.12819E+03
-126	-.12239E+03	-.12158E+03	-.12247E+03	-.12199E+03	-.11943E+03	-.11989E+03	-.13912E+03	-.11152E+03	-.12020E+03	-.11512E+03
-125	-.45861E+02	-.12505E+03	-.12709E+03	-.11967E+03	-.12113E+03	-.12084E+03	-.13357E+03	-.12971E+03	-.12481E+03	-.12210E+03
-124	-.20000E+03	-.11750E+03	-.12189E+03	-.12450E+03	-.12771E+03	-.12041E+03	-.11845E+03	-.12830E+03	-.12182E+03	-.12452E+03
-123	-.45720E+02	-.42711E+02	-.12086E+03	-.12001E+03	-.11976E+03	-.12457E+03	-.13444E+03	-.11711E+03	-.10945E+03	-.12050E+03
-122	-.12026E+03	-.12795E+03	-.11951E+03	-.14285E+03	-.13252E+03	-.12787E+03	-.12099E+03	-.12285E+03	-.11731E+03	-.12192E+03
-121	-.45579E+02	-.13790E+03	-.12608E+03	-.12137E+03	-.11931E+03	-.12189E+03	-.15999E+03	-.11636E+03	-.13510E+03	-.12777E+03
-120	-.20000E+03	-.20000E+03	-.11870E+03	-.12087E+03	-.12107E+03	-.12715E+03	-.14736E+03	-.12688E+03	-.12766E+03	-.11374E+03

K	$P_K(1-B)$	$P_K(2-B)$	$P_K(3-B)$	$P_K(4-B)$	$P_K(5-B)$	$P_K(6-B)$	$P_K(7-B)$	$P_K(8-B)$	$P_K(9-B)$	$P_K(10-B)$
-119	-.45432E+02	-.42423E+02	-.41737E+02	-.12305E+03	-.11608E+03	-.11732E+03	-.13041E+03	-.12857E+03	-.11263E+03	-.14795E+03
-118	-.11845E+03	-.11789E+03	-.13171E+03	-.12405E+03	-.12748E+03	-.11814E+03	-.14416E+03	-.12673E+03	-.12845E+03	-.12519E+03
-117	-.45287E+02	-.12065E+03	-.14785E+03	-.12122E+03	-.14790E+03	-.12406E+03	-.12084E+03	-.12340E+03	-.11945E+03	-.12344E+03
-116	-.20000E+03	-.11482E+03	-.14071E+03	-.12701E+03	-.12246E+03	-.11697E+03	-.11563E+03	-.13473E+03	-.13470E+03	-.11572E+03
-115	-.45135E+02	-.42127E+02	-.12511E+03	-.15601E+03	-.13167E+03	-.11811E+03	-.11571E+03	-.11812E+03	-.11724E+03	-.12663E+03
-114	-.11685E+03	-.14125E+03	-.12431E+03	-.12776E+03	-.11555E+03	-.13216E+03	-.12031E+03	-.12575E+03	-.12987E+03	-.12113E+03
-113	-.44985E+02	-.13619E+03	-.12666E+03	-.12345E+03	-.13509E+03	-.11876E+03	-.12195E+03	-.11739E+03	-.11889E+03	-.13411E+03
-112	-.20000E+03	-.20000E+03	-.20000E+03	-.11687E+03	-.12582E+03	-.12845E+03	-.16805E+03	-.13059E+03	-.12631E+03	-.12325E+03
-111	-.44828E+02	-.41818E+02	-.41130E+02	-.40963E+02	-.12185E+03	-.12987E+03	-.13844E+03	-.11176E+03	-.11379E+03	-.13002E+03
-110	-.11542E+03	-.12286E+03	-.11884E+03	-.12429E+03	-.11546E+03	-.11503E+03	-.15199E+03	-.13859E+03	-.12092E+03	-.13121E+03
-109	-.44672E+02	-.11739E+03	-.11588E+03	-.12737E+03	-.13497E+03	-.12890E+03	-.12044E+03	-.12471E+03	-.11536E+03	-.16312E+03
-108	-.20000E+03	-.12061E+03	-.11974E+03	-.12590E+03	-.15436E+03	-.12503E+03	-.10265E+03	-.13449E+03	-.12787E+03	-.12217E+03
-107	-.44510E+02	-.41501E+02	-.12412E+03	-.12245E+03	-.12501E+03	-.12956E+03	-.11959E+03	-.11080E+03	-.12019E+03	-.12206E+03
-106	-.12214E+03	-.14057E+03	-.12112E+03	-.13292E+03	-.11921E+03	-.11954E+03	-.11407E+03	-.11442E+03	-.11444E+03	-.12333E+03
-105	-.44347E+02	-.13128E+03	-.11634E+03	-.12386E+03	-.12425E+03	-.11684E+03	-.11472E+03	-.13173E+03	-.11039E+03	-.12744E+03
-104	-.20000E+03	-.20000E+03	-.12287E+03	-.12338E+03	-.12295E+03	-.12103E+03	-.14229E+03	-.12141E+03	-.11866E+03	-.11923E+03
-103	-.44179E+02	-.41168E+02	-.40481E+02	-.13057E+03	-.11672E+03	-.12041E+03	-.12433E+03	-.12042E+03	-.12774E+03	-.11685E+03
-102	-.12669E+03	-.11896E+03	-.12513E+03	-.11879E+03	-.12128E+03	-.12339E+03	-.12484E+03	-.12089E+03	-.11652E+03	-.12950E+03
-101	-.44009E+02	-.12878E+03	-.13161E+03	-.12283E+03	-.12669E+03	-.12831E+03	-.11786E+03	-.12612E+03	-.11389E+03	-.12708E+03
-100	-.20000E+03	-.11795E+03	-.14400E+03	-.11982E+03	-.11964E+03	-.11472E+03	-.13889E+03	-.11156E+03	-.11966E+03	-.12512E+03
-99	-.43835E+02	-.40825E+02	-.12770E+03	-.12490E+03	-.11610E+03	-.12329E+03	-.12823E+03	-.11697E+03	-.13439E+03	-.12196E+03
-98	-.12576E+03	-.12800E+03	-.12688E+03	-.13338E+03	-.13224E+03	-.11815E+03	-.12940E+03	-.11259E+03	-.11811E+03	-.11919E+03
-97	-.43658E+02	-.14005E+03	-.13268E+03	-.12981E+03	-.12857E+03	-.12331E+03	-.13110E+03	-.11594E+03	-.11358E+03	-.13249E+03
-96	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12145E+03	-.12254E+03	-.13007E+03	-.12156E+03	-.11910E+03	-.11681E+03
-95	-.43476E+02	-.40466E+02	-.39778E+02	-.39610E+02	-.39569E+02	-.12086E+03	-.12076E+03	-.11683E+03	-.13061E+03	-.14116E+03
-94	-.12485E+03	-.11777E+03	-.12342E+03	-.12116E+03	-.13153E+03	-.13164E+03	-.12429E+03	-.11505E+03	-.12810E+03	-.12369E+03
-93	-.43292E+02	-.12182E+03	-.12265E+03	-.11840E+03	-.13072E+03	-.11091E+03	-.12816E+03	-.12846E+03	-.13088E+03	-.10698E+03
-92	-.20000E+03	-.11457E+03	-.11883E+03	-.12373E+03	-.12155E+03	-.12032E+03	-.11556E+03	-.11268E+03	-.11682E+03	-.11849E+03
-91	-.43103E+02	-.40093E+02	-.11741E+03	-.14242E+03	-.11925E+03	-.12520E+03	-.13033E+03	-.11214E+03	-.11475E+03	-.12157E+03
-90	-.12397E+03	-.12984E+03	-.12757E+03	-.13261E+03	-.12230E+03	-.11686E+03	-.13402E+03	-.11741E+03	-.12203E+03	-.12236E+03
-89	-.42911E+02	-.14116E+03	-.11959E+03	-.12345E+03	-.12033E+03	-.11945E+03	-.12252E+03	-.12452E+03	-.15397E+03	-.12071E+03
-88	-.20000E+03	-.20000E+03	-.11504E+03	-.11785E+03	-.13773E+03	-.11572E+03	-.14800E+03	-.11449E+03	-.12604E+03	-.12074E+03
-87	-.42712E+02	-.39702E+02	-.59015E+02	-.11911E+03	-.12815E+03	-.11793E+03	-.12374E+03	-.12415E+03	-.13147E+03	-.12051E+03
-86	-.12309E+03	-.12464E+03	-.13150E+03	-.11910E+03	-.12252E+03	-.12372E+03	-.12459E+03	-.13907E+03	-.11598E+03	-.11457E+03
-85	-.42511E+02	-.12498E+03	-.13715E+03	-.12875E+03	-.13664E+03	-.14651E+03	-.11874E+03	-.11108E+03	-.11103E+03	-.11743E+03
-84	-.20000E+03	-.12413E+03	-.14433E+03	-.12009E+03	-.11765E+03	-.13143E+03	-.12032E+03	-.12373E+03	-.13526E+03	-.14051E+03
-83	-.42303E+02	-.39294E+02	-.13487E+03	-.12796E+03	-.13529E+03	-.11659E+03	-.12785E+03	-.12470E+03	-.13169E+03	-.12029E+03
-82	-.12098E+03	-.13882E+03	-.12853E+03	-.12992E+03	-.12297E+03	-.12607E+03	-.12221E+03	-.11701E+03	-.13071E+03	-.12222E+03
-81	-.42092E+02	-.13880E+03	-.13525E+03	-.13274E+03	-.13721E+03	-.13369E+03	-.12864E+03	-.12817E+03	-.13196E+03	-.11734E+03
-80	-.14433E+03	-.14434E+03	-.14683E+03	-.12570E+03	-.14028E+03	-.12147E+03	-.12724E+03	-.12569E+03	-.12798E+03	-.13308E+03
-79	-.41875E+02	-.38864E+02	-.38177E+02	-.38009E+02	-.13287E+03	-.13305E+03	-.12384E+03	-.11651E+03	-.11406E+03	-.11212E+03
-78	-.12527E+03	-.12069E+03	-.12216E+03	-.12764E+03	-.12650E+03	-.12781E+03	-.12152E+03	-.12765E+03	-.13039E+03	-.12466E+03
-77	-.41653E+02	-.12723E+03	-.12607E+03	-.14082E+03	-.11995E+03	-.14372E+03	-.11626E+03	-.12528E+03	-.12830E+03	-.12885E+03
-76	-.20000E+03	-.12573E+03	-.12412E+03	-.15397E+03	-.12901E+03	-.13531E+03	-.12097E+03	-.12815E+03	-.11638E+03	-.12752E+03
-75	-.41422E+02	-.38413E+02	-.12989E+03	-.12739E+03	-.13354E+03	-.14161E+03	-.12529E+03	-.12792E+03	-.15801E+03	-.12651E+03
-74	-.12515E+03	-.20000E+03	-.13674E+03	-.12767E+03	-.12099E+03	-.12850E+03	-.12455E+03	-.11732E+03	-.11817E+03	-.12603E+03
-73	-.41189E+02	-.14258E+03	-.13167E+03	-.14347E+03	-.12301E+03	-.14090E+03	-.12362E+03	-.15249E+03	-.12905E+03	-.12674E+03
-72	-.14342E+03	-.20000E+03	-.12682E+03	-.13287E+03	-.12807E+03	-.12194E+03	-.12959E+03	-.13452E+03	-.12170E+03	-.12173E+03
-71	-.40947E+02	-.37937E+02	-.37250E+02	-.13443E+03	-.13046E+03	-.13384E+03	-.13085E+03	-.12203E+03	-.12407E+03	-.15504E+03
-70	-.12616E+03	-.12309E+03	-.12890E+03	-.12578E+03	-.14141E+03	-.13414E+03	-.12716E+03	-.12185E+03	-.11728E+03	-.13397E+03
-69	-.40700E+02	-.12705E+03	-.14213E+03	-.13400E+03	-.12993E+03	-.14066E+03	-.12201E+03	-.13701E+03	-.12545E+03	-.14558E+03
-68	-.20000E+03	-.12676E+03	-.13143E+03	-.14718E+03	-.13322E+03	-.13804E+03	-.12105E+03	-.12628E+03	-.13925E+03	-.12649E+03
-67	-.40444E+02	-.37434E+02	-.13903E+03	-.14516E+03	-.12871E+03	-.13410E+03	-.11847E+03	-.12797E+03	-.12071E+03	-.12968E+03
-66	-.12606E+03	-.12866E+03	-.13080E+03	-.13175E+03	-.12934E+03	-.13164E+03	-.13245E+03	-.13551E+03	-.12489E+03	-.12922E+03
-65	-.40181E+02	-.13645E+03	-.13993E+03	-.13405E+03	-.13076E+03	-.12990E+03	-.12949E+03	-.12200E+03	-.12519E+03	-.12396E+03
-64	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12584E+03	-.12247E+03	-.12183E+03	-.11909E+03	-.12968E+03
-63	-.39909E+02	-.36899E+02	-.36211E+02	-.36043E+02	-.36001E+02	-.35990E+02	-.13287E+03	-.12532E+03	-.12553E+03	-.14292E+03
-62	-.12595E+03	-.12597E+03	-.12388E+03	-.12150E+03	-.12104E+03	-.12779E+03	-.11571E+03	-.12074E+03	-.11419E+03	-.12321E+03
-61	-.39629E+02	-.12844E+03	-.12264E+03	-.12350E+03	-.11839E+03	-.13507E+03	-.12579E+03	-.12594E+03	-.12354E+03	-.12231E+03
-60	-.14184E+03	-.12522E+03	-.12449E+03	-.12856E+03	-.11801E+03	-.14741E+03	-.11757E+03	-.12430E+03	-.12171E+03	-.12841E+03
-59	-.39339E+02	-.36329E+02	-.12832E+03	-.14347E+03	-.12238E+03	-.14609E+03	-.12090E+03	-.11787E+03	-.14467E+03	-.14847E+03
-58	-.12583E+03	-.13551E+03	-.13067E+03	-.14300E+03	-.12186E+03	-.12998E+03	-.11976E+03	-.14699E+03	-.15573E+03	-.12079E+03
-57	-.39040E+02	-.13698E+03	-.12810E+03	-.12623E+03	-.11975E+03	-.13130E+03	-.13143E+03	-.12945E+03	-.12187E+03	-.12753E+03
-56	-.20000E+03	-.14124E+03	-.12506E+03	-.12169E+03	-.13611E+03	-.13673E+03	-.11647E+03	-.12933E+03	-.13958E+03	-.12200E+03
-55	-.38729E+02	-.35719E+02	-.35032E+02	-.13193E+03	-.12511E+03	-.15757E+03	-.11821E+03	-.13226E+03	-.13424E+03	-.13149E+03
-54	-.12734E+03	-.11951E+03	-.13143E+03	-.14775E+03	-.13096E+03	-.13028E+03	-.12979E+03	-.12034E+03	-.13447E+03	-.12921E+03
-53	-.38408E+02	-.12643E+03	-.14859E+03	-.13287E+03	-.11797E+03	-.16130E+03	-.12966E+03	-.12992E+03	-.14495E+03	-.12764E+03
-52	-.14059E+03	-.12355E+03	-.13571E+03	-.12423E+03	-.12372E+03	-.14520E+03	-.12664E+03	-.13887E+03	-.12215E+03	-.12384E+03
-51	-.38074E+02	-.35064E+02	-.13611E+03	-.12634E+03	-.12247E+03	-.13691E+03	-.13062E+03	-.12770E+03	-.13402E+03	-.12793E+03
-50	-.12406E+03	-.16858E+03	-.13009E+03	-.13291E+03	-.12523E+03	-.13875E+03	-.13096E+03	-.12087E+03	-.11833E+03	-.12789E+03
-49	-.37727E+02	-.14134E+03	-.13149E+03	-.12976E+03	-.12939E+03	-.12736E+03	-.13030E+03	-.13285E+03	-.11379E+03	-.13242E+03
-48	-.13990E+03	-.20000E+03	-.15194E+03	-.12653E+03	-.12197E+03	-.11733E+03	-.13030E+03	-.12705E+03	-.12473E+03	-.12830E+03
-47	-.37364E+02	-.36354E+02	-.33666E+02	-.33498E+02	-.13108E+03	-.13933E+03	-.12547E+03	-.12788E+03	-.12215E+03	-.12059E+03
-46	-.12380E+03	-.12260E+03	-.12123E+03	-.12800E+03	-.12958E+03	-.11446E+03	-.11958E+03	-.13219E+03	-.11815E+03	-.10502E+03
-45	-.36987E+02	-.12483E+03	-.12397E+03	-.13676E+03	-.13180E+03	-.13030E+03	-.11617E+03	-.12401E+03	-.12173E+03	-.12792E+03
-44	-.20000E+03	-.12617E+03	-.12553E+03	-.14347E+03	-.13745E+03	-.11906E+03	-.11698E+03	-.12429E+03	-.11801E+03	-.12550E+03
-43	-.36592E+02	-.33582E+02	-.12949E+03	-.12572E+03	-.12630E+03	-.12269E+03	-.13721E+03	-.12613E+03	-.12168E+03	-.12212E+03
-42	-.12349E+03	-.12678E+03	-.13259E+03	-.12893E+03	-.13020E+03	-.11909E+03	-.12840E+03	-.12328E+03	-.13356E+03	-.12357E+03
-41	-.36178E+02	-.13393E+03	-.12785E+03	-.13654E+03	-.13206E+03	-.12699E+03	-.12969E+03	-.12092E+03	-.11788E+03	-.12773E+03
-40	-.13831E+03	-.14081E+03	-.12391E+03	-.12949E+03	-.12213E+03	-.12387E+03	-.12048E+03	-.12136E+03	-.13858E+03	-.12752E+03

K	$P_K(1-B)$	$P_K(2-B)$	$P_K(3-B)$	$P_K(4-B)$	$P_K(5-B)$	$P_K(6-B)$	$P_K(7-B)$	$P_K(8-B)$	$P_K(9-B)$	$P_K(10-B)$
-39	-.35744E+02	-.32733E+02	-.32046E+02	-.13193E+03	-.12597E+03	-.11998E+03	-.12767E+03	-.11626E+03	-.11291E+03	-.13047E+03
-38	-.12489E+03	-.12323E+03	-.12397E+03	-.12752E+03	-.12979E+03	-.12101E+03	-.11647E+03	-.13902E+03	-.13762E+03	-.12448E+03
-37	-.35287E+02	-.12783E+03	-.13073E+03	-.13009E+03	-.13721E+03	-.12840E+03	-.12264E+03	-.13267E+03	-.12465E+03	-.12533E+03
-36	-.20000E+03	-.12379E+03	-.13167E+03	-.13903E+03	-.12675E+03	-.12721E+03	-.11856E+03	-.12434E+03	-.13684E+03	-.12365E+03
-35	-.34804E+02	-.31793E+02	-.13393E+03	-.13795E+03	-.12981E+03	-.12553E+03	-.11805E+03	-.12579E+03	-.12599E+03	-.12503E+03
-34	-.12458E+03	-.13875E+03	-.13301E+03	-.13151E+03	-.13056E+03	-.12123E+03	-.11872E+03	-.12283E+03	-.12932E+03	-.12608E+03
-33	-.34293E+02	-.16256E+03	-.13202E+03	-.13349E+03	-.12953E+03	-.12494E+03	-.12256E+03	-.12688E+03	-.12864E+03	-.12792E+03
-32	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12414E+03	-.12553E+03	-.12930E+03	-.12755E+03	-.13020E+03	-.12759E+03
-31	-.33749E+02	-.30739E+02	-.30052E+02	-.29883E+02	-.29841E+02	-.13062E+03	-.12387E+03	-.14017E+03	-.12972E+03	-.13151E+03
-30	-.12422E+03	-.12357E+03	-.12209E+03	-.12018E+03	-.13655E+03	-.11903E+03	-.13795E+03	-.12324E+03	-.12474E+03	-.12477E+03
-29	-.33171E+02	-.12511E+03	-.12105E+03	-.12273E+03	-.14011E+03	-.12893E+03	-.15051E+03	-.13735E+03	-.13154E+03	-.13552E+03
-28	-.13522E+03	-.12529E+03	-.12149E+03	-.12866E+03	-.16256E+03	-.11427E+03	-.13069E+03	-.13069E+03	-.12733E+03	-.12172E+03
-27	-.32549E+02	-.29539E+02	-.12744E+03	-.15167E+03	-.12832E+03	-.11587E+03	-.12578E+03	-.12871E+03	-.12902E+03	-.12794E+03
-26	-.12376E+03	-.13676E+03	-.12858E+03	-.13770E+03	-.13745E+03	-.12499E+03	-.12692E+03	-.12570E+03	-.12941E+03	-.12590E+03
-25	-.31881E+02	-.13933E+03	-.12643E+03	-.12197E+03	-.13096E+03	-.12494E+03	-.12282E+03	-.13227E+03	-.14258E+03	-.12761E+03
-24	-.20000E+03	-.14592E+03	-.12531E+03	-.11948E+03	-.14630E+03	-.11829E+03	-.11676E+03	-.12402E+03	-.12772E+03	-.12250E+03
-23	-.31157E+02	-.28147E+02	-.27459E+02	-.12875E+03	-.12911E+03	-.12048E+03	-.12181E+03	-.13660E+03	-.13154E+03	-.13482E+03
-22	-.12650E+03	-.12153E+03	-.12744E+03	-.15658E+03	-.12477E+03	-.14097E+03	-.12529E+03	-.13316E+03	-.12788E+03	-.12270E+03
-21	-.30367E+02	-.12165E+03	-.12893E+03	-.13551E+03	-.13676E+03	-.14565E+03	-.12157E+03	-.12024E+03	-.12767E+03	-.12515E+03
-20	-.13831E+03	-.12358E+03	-.16858E+03	-.12108E+03	-.12921E+03	-.13062E+03	-.12397E+03	-.12382E+03	-.13535E+03	-.12226E+03
-19	-.29497E+02	-.26487E+02	-.13551E+03	-.12392E+03	-.14173E+03	-.12940E+03	-.12333E+03	-.11589E+03	-.18664E+03	-.13905E+03
-18	-.11975E+03	-.13654E+03	-.12664E+03	-.12714E+03	-.16958E+03	-.12729E+03	-.12024E+03	-.14323E+03	-.14102E+03	-.12914E+03
-17	-.28531E+02	-.14213E+03	-.12752E+03	-.13340E+03	-.11945E+03	-.13009E+03	-.14949E+03	-.12224E+03	-.12503E+03	-.12557E+03
-16	-.20000E+03	-.20000E+03	-.20000E+03	-.12149E+03	-.12832E+03	-.15903E+03	-.12273E+03	-.12825E+03	-.13178E+03	-.13615E+03
-15	-.27444E+02	-.24434E+02	-.23746E+02	-.23578E+02	-.13219E+03	-.13654E+03	-.14630E+03	-.12865E+03	-.13229E+03	-.13142E+03
-14	-.11835E+03	-.12116E+03	-.12045E+03	-.13552E+03	-.12783E+03	-.12547E+03	-.11605E+03	-.12461E+03	-.12565E+03	-.13462E+03
-13	-.26201E+02	-.12439E+03	-.11660E+03	-.14256E+03	-.11936E+03	-.12729E+03	-.12466E+03	-.12159E+03	-.12990E+03	-.12446E+03
-12	-.20000E+03	-.11885E+03	-.13193E+03	-.13460E+03	-.12020E+03	-.12824E+03	-.11649E+03	-.12508E+03	-.12821E+03	-.13455E+03
-11	-.24750E+02	-.21740E+02	-.12116E+03	-.11915E+03	-.12086E+03	-.13167E+03	-.13571E+03	-.12304E+03	-.13333E+03	-.12498E+03
-10	-.11779E+03	-.14775E+03	-.12247E+03	-.14959E+03	-.12260E+03	-.12423E+03	-.13947E+03	-.13541E+03	-.13039E+03	-.13614E+03
-9	-.23007E+02	-.13611E+03	-.12340E+03	-.15903E+03	-.12902E+03	-.15460E+03	-.12930E+03	-.12749E+03	-.13073E+03	-.16914E+03
-8	-.20000E+03	-.20000E+03	-.11783E+03	-.15301E+03	-.13119E+03	-.11970E+03	-.14949E+03	-.12866E+03	-.13247E+03	-.12300E+03
-7	-.20824E+02	-.17814E+02	-.17126E+02	-.13095E+03	-.14565E+03	-.13041E+03	-.12630E+03	-.12934E+03	-.12917E+03	-.12584E+03
-6	-.11661E+03	-.12407E+03	-.12591E+03	-.12565E+03	-.12058E+03	-.12181E+03	-.12230E+03	-.12528E+03	-.12747E+03	-.12984E+03
-5	-.17902E+02	-.11654E+03	-.12807E+03	-.12021E+03	-.12455E+03	-.12185E+03	-.12243E+03	-.12222E+03	-.11874E+03	-.12113E+03
-4	-.20000E+03	-.12603E+03	-.13847E+03	-.13963E+03	-.12875E+03	-.13517E+03	-.12979E+03	-.13179E+03	-.12289E+03	-.12346E+03
-3	-.13465E+02	-.10455E+02	-.13245E+03	-.15167E+03	-.12517E+03	-.12671E+03	-.12685E+03	-.12942E+03	-.13084E+03	-.12861E+03
-2	-.12444E+03	-.13245E+03	-.13245E+03	-.12439E+03	-.12352E+03	-.13206E+03	-.12466E+03	-.13508E+03	-.12129E+03	-.11729E+03
-1	-.39224E+01	-.13051E+03	-.13245E+03	-.13062E+03	-.12875E+03	-.12500E+03	-.12500E+03	-.13459E+03	-.13261E+03	-.12089E+03
0	-.20000E+03	-.20000E+03	-.13245E+03	-.13495E+03	-.15051E+03	-.14699E+03	-.11139E+03	-.10671E+03	-.10496E+03	-.10443E+03
1	-.39224E+01	-.91210E+00	-.22441E+00	-.55987E-01	-.13965E-01	-.34927E-02	-.87447E-03	-.21952E-03	-.59539E-04	-.17085E-04
2	-.13398E+03	-.13495E+03	-.13051E+03	-.13301E+03	-.14699E+03	-.14699E+03	-.11139E+03	-.10671E+03	-.10496E+03	-.10443E+03
3	-.13465E+02	-.12759E+03	-.12893E+03	-.13108E+03	-.13795E+03	-.12471E+03	-.12505E+03	-.12447E+03	-.13258E+03	-.12089E+03
4	-.20000E+03	-.13046E+03	-.13245E+03	-.12511E+03	-.12352E+03	-.13155E+03	-.12466E+03	-.13512E+03	-.12128E+03	-.11729E+03
5	-.17902E+02	-.14891E+02	-.13041E+03	-.14630E+03	-.12523E+03	-.12671E+03	-.12678E+03	-.12953E+03	-.13084E+03	-.12861E+03
6	-.12724E+03	-.13131E+03	-.13495E+03	-.13551E+03	-.13770E+03	-.13513E+03	-.12959E+03	-.13197E+03	-.12298E+03	-.12346E+03
7	-.20824E+02	-.13316E+03	-.12076E+03	-.12011E+03	-.12460E+03	-.12189E+03	-.12234E+03	-.12224E+03	-.11874E+03	-.12113E+03
8	-.20000E+03	-.20000E+03	-.11655E+03	-.12407E+03	-.12045E+03	-.12181E+03	-.12226E+03	-.12528E+03	-.12747E+03	-.12983E+03
9	-.23007E+02	-.19997E+02	-.19309E+02	-.12884E+03	-.14300E+03	-.13009E+03	-.12630E+03	-.12917E+03	-.12920E+03	-.12585E+03
10	-.11676E+03	-.12377E+03	-.13287E+03	-.13245E+03	-.13062E+03	-.11966E+03	-.15167E+03	-.13965E+03	-.13250E+03	-.13298E+03
11	-.24750E+02	-.12201E+03	-.15654E+03	-.12979E+03	-.13155E+03	-.15654E+03	-.12930E+03	-.12748E+03	-.13078E+03	-.17183E+03
12	-.20000E+03	-.12660E+03	-.12979E+03	-.13346E+03	-.12291E+03	-.12473E+03	-.13821E+03	-.13336E+03	-.13043E+03	-.13616E+03
13	-.26201E+02	-.23191E+02	-.13847E+03	-.11979E+03	-.12097E+03	-.13155E+03	-.13551E+03	-.12502E+03	-.13329E+03	-.12499E+03
14	-.12399E+03	-.12999E+03	-.14858E+03	-.12959E+03	-.14028E+03	-.12815E+03	-.11656E+03	-.12513E+03	-.12821E+03	-.13453E+03
15	-.27444E+02	-.13995E+03	-.12699E+03	-.12643E+03	-.11979E+03	-.12692E+03	-.12455E+03	-.12162E+03	-.12986E+03	-.12446E+03
16	-.20000E+03	-.20000E+03	-.20000E+03	-.12050E+03	-.12714E+03	-.12529E+03	-.11607E+03	-.12461E+03	-.12565E+03	-.13462E+03
17	-.28531E+02	-.25521E+02	-.24833E+02	-.24665E+02	-.13143E+03	-.13795E+03	-.14397E+03	-.12857E+03	-.13221E+03	-.13143E+03
18	-.12473E+03	-.12001E+03	-.11831E+03	-.12824E+03	-.13193E+03	-.15460E+02	-.12264E+03	-.12835E+03	-.13171E+03	-.13616E+03
19	-.29497E+02	-.12721E+03	-.11957E+03	-.13193E+03	-.11993E+03	-.12969E+03	-.15301E+03	-.12226E+03	-.12503E+03	-.12557E+03
20	-.13831E+03	-.12257E+03	-.12142E+03	-.15522E+03	-.14213E+03	-.12721E+03	-.12038E+03	-.14362E+03	-.14100E+03	-.12915E+03
21	-.30367E+02	-.27356E+02	-.12824E+03	-.12759E+03	-.13477E+03	-.12999E+03	-.12328E+03	-.11593E+03	-.17868E+03	-.13905E+03
22	-.12524E+03	-.14347E+03	-.12866E+03	-.12157E+03	-.12940E+03	-.13020E+03	-.12392E+03	-.12381E+03	-.13531E+03	-.12226E+03
23	-.31157E+02	-.15301E+03	-.13477E+03	-.15167E+03	-.15654E+03	-.14134E+03	-.12191E+03	-.12029E+03	-.12769E+03	-.12513E+03
24	-.20000E+03	-.14592E+03	-.12428E+03	-.14256E+03	-.12439E+03	-.13963E+03	-.12511E+03	-.13302E+03	-.12792E+03	-.12269E+03
25	-.31881E+02	-.28871E+02	-.28183E+02	-.13301E+03	-.12875E+03	-.12062E+03	-.12165E+03	-.13710E+03	-.13158E+03	-.13481E+03
26	-.12296E+03	-.12517E+03	-.12911E+03	-.12193E+03	-.13426E+03	-.11861E+03	-.11671E+03	-.12408E+03	-.12772E+03	-.12250E+03
27	-.32549E+02	-.12309E+03	-.15654E+03	-.12471E+03	-.14213E+03	-.12505E+03	-.12305E+03	-.13225E+03	-.14264E+03	-.12763E+03
28	-.13522E+03	-.12440E+03	-.13513E+03	-.14949E+03	-.14134E+03	-.12483E+03	-.12729E+03	-.12573E+03	-.12944E+03	-.12591E+03
29	-.33170E+02	-.30160E+02	-.13477E+03	-.20000E+03	-.12619E+03	-.11626E+03	-.12559E+03	-.12957E+03	-.12901E+03	-.12793E+03
30	-.12348E+03	-.13119E+03	-.12866E+03	-.13495E+03	-.13291E+03	-.11469E+03	-.15654E+03	-.13076E+03	-.12733E+03	-.12171E+03
31	-.33750E+02	-.14134E+03	-.13051E+03	-.13308E+03	-.12782E+03	-.12999E+03	-.14505E+03	-.13729E+03	-.13155E+03	-.13552E+03
32	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12346E+03	-.11933E+03	-.14062E+03	-.12327E+03	-.12475E+03	-.12478E+03
33	-.34293E+02	-.31283E+02	-.30595E+02	-.30426E+02	-.30384E+02	-.12089E+03	-.12444E+03	-.13940E+03	-.12963E+03	-.13151E+03
34	-.12390E+03	-.12439E+03	-.11815E+03	-.11990E+03	-.12956E+03	-.12553E+03	-.13020E+03	-.12765E+03	-.13019E+03	-.12757E+03
35	-.34804E+02	-.12591E+03	-.12762E+03	-.12483E+03	-.12891E+03	-.12511E+03	-.12296E+03	-.12693E+03	-.12865E+03	-.12792E+03
36	-.20000E+03	-.12320E+03	-.12650E+03	-.12572E+03	-.14601E+03	-.12165E+03	-.11899E+03	-.12089E+03	-.12926E+03	-.12609E+03
37	-.25287E+02	-.32276E+02	-.12685E+03	-.13287E+03	-.13933E+03	-.12578E+03	-.11809E+03	-.12565E+03	-.12600E+03	-.12503E+03
38	-.12425E+03	-.16858E+03	-.14347E+03	-.14256E+03	-.12759E+03	-.12729E+03	-.11883E+03	-.12440E+03	-.13676E+03	-.12366E+03
39	-.35743E+02	-.20000E+03	-.13076E+03	-.12911E+03	-.15301E+03	-.12791E+03	-.12319E+03	-.13271E+03	-.12467E+03	-.12533E+03
40	-.13831E+03	-.14081E+03	-.12723E+03	-.12578E+03	-.16256E+03	-.12238E+03	-.11682E+03	-.13894E+03	-.13765E+03	-.12448E+03

K	$P_K(1-B)$	$P_K(2-B)$	$P_K(3-B)$	$P_K(4-B)$	$P_K(5-B)$	$P_K(6-B)$	$P_K(7-B)$	$P_K(8-B)$	$P_K(9-B)$	$P_K(10-B)$
41	-.36178E+02	-.33168E+02	-.32480E+02	-.12902E+03	-.12685E+03	-.12112E+03	-.12866E+03	-.11636E+03	-.11292E+03	-.13048E+03
42	-.12300E+03	-.11785E+03	-.12402E+03	-.13443E+03	-.12260E+03	-.12523E+03	-.12028E+03	-.12138E+03	-.13849E+03	-.12752E+03
43	-.36592E+02	-.12799E+03	-.16256E+03	-.13143E+03	-.13460E+03	-.12875E+03	-.12930E+03	-.12097E+03	-.11790E+03	-.12774E+03
44	-.20000E+03	-.12554E+03	-.13611E+03	-.12671E+03	-.13245E+03	-.11966E+03	-.12807E+03	-.12339E+03	-.13355E+03	-.12357E+03
45	-.36987E+02	-.33976E+02	-.13847E+03	-.12553E+03	-.12471E+03	-.12623E+03	-.13847E+03	-.12659E+03	-.12168E+03	-.12213E+03
46	-.12655E+03	-.12979E+03	-.13273E+03	-.12997E+03	-.14134E+03	-.12048E+03	-.11728E+03	-.12433E+03	-.11803E+03	-.12551E+03
47	-.37364E+02	-.14062E+03	-.13405E+03	-.13236E+03	-.12999E+03	-.12969E+03	-.11615E+03	-.12407E+03	-.12174E+03	-.12792E+03
48	-.13990E+03	-.20000E+03	-.15194E+03	-.12593E+03	-.13020E+03	-.11530E+03	-.11847E+03	-.13231E+03	-.11816E+03	-.10502E+03
49	-.37727E+02	-.34716E+02	-.34028E+02	-.33860E+02	-.13331E+03	-.13611E+03	-.12500E+03	-.12760E+03	-.12211E+03	-.12058E+03
50	-.12666E+03	-.12494E+03	-.12169E+03	-.12710E+03	-.12305E+03	-.11912E+03	-.13167E+03	-.12714E+03	-.12474E+03	-.12829E+03
51	-.38074E+02	-.12678E+03	-.12238E+03	-.13893E+03	-.12979E+03	-.13049E+03	-.13377E+03	-.13377E+03	-.11379E+03	-.13260E+03
52	-.14059E+03	-.12587E+03	-.12500E+03	-.15167E+03	-.12657E+03	-.13573E+03	-.13180E+03	-.12093E+03	-.11881E+03	-.12788E+03
53	-.38408E+02	-.35398E+02	-.12832E+03	-.12759E+03	-.12460E+03	-.13450E+03	-.13041E+03	-.12740E+03	-.13405E+03	-.12795E+03
54	-.12675E+03	-.12535E+03	-.13301E+03	-.12529E+03	-.12535E+03	-.13877E+03	-.12603E+03	-.13922E+03	-.12216E+03	-.12384E+03
55	-.38729E+02	-.13632E+03	-.12914E+03	-.13821E+03	-.11966E+03	-.14368E+03	-.12999E+03	-.12993E+03	-.14502E+03	-.12763E+03
56	-.20000E+03	-.14124E+03	-.12462E+03	-.13591E+03	-.13346E+03	-.12915E+03	-.12893E+03	-.12041E+03	-.13450E+03	-.12924E+03
57	-.39040E+02	-.36030E+02	-.35342E+02	-.13571E+03	-.12752E+03	-.14007E+03	-.11810E+03	-.13177E+03	-.13425E+03	-.13149E+03
58	-.12536E+03	-.12119E+03	-.13145E+03	-.12357E+03	-.13954E+03	-.13246E+03	-.11682E+03	-.12946E+03	-.13965E+03	-.12199E+03
59	-.39339E+02	-.12705E+03	-.13495E+03	-.12930E+03	-.12268E+03	-.12955E+03	-.12959E+03	-.12951E+03	-.12189E+03	-.12754E+03
60	-.14184E+03	-.12480E+03	-.14630E+03	-.15922E+03	-.12555E+03	-.14565E+03	-.11976E+03	-.14660E+03	-.13577E+03	-.12079E+03
61	-.39629E+02	-.36619E+02	-.13591E+03	-.15724E+03	-.12707E+03	-.13658E+03	-.12090E+03	-.11804E+03	-.14477E+03	-.14847E+03
62	-.12551E+03	-.14213E+03	-.13005E+03	-.13478E+03	-.12405E+03	-.13439E+03	-.11813E+03	-.12429E+03	-.12173E+03	-.12838E+03
63	-.39909E+02	-.14406E+03	-.13201E+03	-.13329E+03	-.12816E+03	-.13350E+03	-.12678E+03	-.12605E+03	-.12355E+03	-.13234E+03
64	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12542E+03	-.11571E+03	-.12082E+03	-.11420E+03	-.12521E+03
65	-.40181E+02	-.37171E+02	-.36483E+02	-.36514E+02	-.36272E+02	-.36262E+02	-.13119E+03	-.12569E+03	-.12249E+03	-.14292E+03
66	-.12564E+03	-.11973E+03	-.12048E+03	-.12099E+03	-.11956E+03	-.12567E+03	-.12300E+03	-.13201E+03	-.11910E+03	-.12969E+03
67	-.40444E+02	-.12597E+03	-.12643E+03	-.12507E+03	-.12159E+03	-.15093E+03	-.13051E+03	-.12210E+03	-.12519E+03	-.12396E+03
68	-.20000E+03	-.12632E+03	-.12444E+03	-.12599E+03	-.12374E+03	-.14655E+03	-.13392E+03	-.13584E+03	-.12481E+03	-.12921E+03
69	-.40700E+02	-.37689E+02	-.13020E+03	-.13770E+03	-.12472E+03	-.15232E+03	-.11895E+03	-.12751E+03	-.12073E+03	-.12968E+03
70	-.12576E+03	-.13119E+03	-.14156E+03	-.15902E+03	-.13065E+03	-.15072E+03	-.12193E+03	-.12638E+03	-.13931E+03	-.12649E+03
71	-.40947E+02	-.14168E+03	-.13502E+03	-.13051E+03	-.12716E+03	-.15012E+03	-.12291E+03	-.13687E+03	-.12547E+03	-.14558E+03
72	-.14342E+03	-.20000E+03	-.12640E+03	-.12488E+03	-.13803E+03	-.14163E+03	-.12875E+03	-.12196E+03	-.11730E+03	-.13396E+03
73	-.41189E+02	-.38179E+02	-.37491E+02	-.13131E+03	-.12849E+03	-.13933E+03	-.13301E+03	-.12187E+03	-.12410E+03	-.15504E+03
74	-.12481E+03	-.12477E+03	-.12900E+03	-.14134E+03	-.12729E+03	-.12151E+03	-.13073E+03	-.13447E+03	-.12171E+03	-.12173E+03
75	-.41423E+02	-.12748E+03	-.13477E+03	-.13513E+03	-.12185E+03	-.16974E+03	-.12428E+03	-.15377E+03	-.12907E+03	-.12675E+03
76	-.20000E+03	-.12774E+03	-.14097E+03	-.12666E+03	-.11993E+03	-.12909E+03	-.12529E+03	-.11741E+03	-.11819E+03	-.12604E+03
77	-.41653E+02	-.38642E+02	-.13832E+03	-.12850E+03	-.13426E+03	-.15428E+03	-.12603E+03	-.12749E+03	-.15785E+03	-.12652E+03
78	-.12711E+03	-.12866E+03	-.13085E+03	-.13338E+03	-.12969E+03	-.13771E+03	-.12071E+03	-.12932E+03	-.11640E+03	-.12753E+03
79	-.41875E+02	-.13814E+03	-.13792E+03	-.13044E+03	-.11942E+03	-.13774E+03	-.11628E+03	-.12541E+03	-.12831E+03	-.12886E+03
80	-.14433E+03	-.14433E+03	-.14684E+03	-.12759E+03	-.12591E+03	-.12607E+03	-.12132E+03	-.12765E+03	-.13039E+03	-.12466E+03
81	-.42092E+02	-.39082E+02	-.38394E+02	-.38225E+02	-.13131E+03	-.13508E+03	-.12509E+03	-.11672E+03	-.11411E+03	-.11213E+03
82	-.12200E+03	-.12134E+03	-.12201E+03	-.13094E+03	-.13609E+03	-.12139E+03	-.12717E+03	-.12571E+03	-.12798E+03	-.13309E+03
83	-.42303E+02	-.12582E+03	-.12607E+03	-.13688E+03	-.14467E+03	-.13161E+03	-.12985E+03	-.12088E+03	-.13155E+03	-.11746E+03
84	-.20000E+03	-.12387E+03	-.12264E+03	-.14090E+03	-.12448E+03	-.12911E+03	-.13985E+03	-.11778E+03	-.13118E+03	-.12208E+03
85	-.42511E+02	-.39501E+02	-.12815E+03	-.13002E+03	-.13877E+03	-.11768E+03	-.13135E+03	-.12326E+03	-.13216E+03	-.12016E+03
86	-.12286E+03	-.14531E+03	-.13383E+03	-.12520E+03	-.11859E+03	-.12769E+03	-.11589E+03	-.12240E+03	-.14403E+03	-.14234E+03
87	-.42712E+02	-.14224E+03	-.13033E+03	-.11989E+03	-.13842E+03	-.13245E+03	-.11509E+03	-.11152E+03	-.11109E+03	-.11755E+03
88	-.20000E+03	-.20000E+03	-.11445E+03	-.12739E+03	-.12433E+03	-.12739E+03	-.14038E+03	-.13251E+03	-.11608E+03	-.11451E+03
89	-.42911E+02	-.39900E+02	-.39210E+02	-.12513E+03	-.13118E+03	-.11886E+03	-.11763E+03	-.12052E+03	-.13199E+03	-.12069E+03
90	-.12372E+03	-.12600E+03	-.13627E+03	-.12204E+03	-.11698E+03	-.11670E+03	-.12386E+03	-.11511E+03	-.12633E+03	-.12091E+03
91	-.43103E+02	-.12608E+03	-.11941E+03	-.12997E+03	-.11939E+03	-.11848E+03	-.14413E+03	-.12306E+03	-.14066E+03	-.12057E+03
92	-.20000E+03	-.12578E+03	-.12624E+03	-.15045E+03	-.12055E+03	-.11786E+03	-.12136E+03	-.11675E+03	-.12190E+03	-.12216E+03
93	-.43292E+02	-.40082E+02	-.12958E+03	-.13893E+03	-.11893E+03	-.12383E+03	-.12791E+03	-.11267E+03	-.11483E+03	-.12178E+03
94	-.12459E+03	-.13177E+03	-.12846E+03	-.13223E+03	-.12025E+03	-.11929E+03	-.12062E+03	-.11149E+03	-.11694E+03	-.11839E+03
95	-.43477E+02	-.13733E+03	-.13219E+03	-.12854E+03	-.12209E+03	-.11152E+03	-.11981E+03	-.13178E+03	-.13141E+03	-.10695E+03
96	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12031E+03	-.13626E+03	-.11792E+03	-.11576E+03	-.12580E+03	-.12296E+03
97	-.43658E+02	-.40648E+02	-.39960E+02	-.39792E+02	-.39743E+02	-.12241E+03	-.11632E+03	-.11771E+03	-.12109E+03	-.14283E+03
98	-.12548E+03	-.11898E+03	-.11998E+03	-.12145E+03	-.13037E+03	-.12377E+03	-.12915E+03	-.12043E+03	-.11925E+03	-.11672E+03
99	-.43835E+02	-.12491E+03	-.12059E+03	-.11967E+03	-.12742E+03	-.12509E+03	-.12026E+03	-.11677E+03	-.11368E+03	-.13323E+03
100	-.20000E+03	-.11721E+03	-.12241E+03	-.12458E+03	-.11784E+03	-.11891E+03	-.11970E+03	-.11425E+03	-.11824E+03	-.11907E+03
101	-.44009E+02	-.40998E+02	-.12857E+03	-.12079E+03	-.11926E+03	-.12478E+03	-.12018E+03	-.11634E+03	-.13003E+03	-.12214E+03
102	-.12639E+03	-.12843E+03	-.11899E+03	-.11735E+03	-.12631E+03	-.11565E+03	-.12164E+03	-.11209E+03	-.11982E+03	-.12545E+03
103	-.44179E+02	-.14106E+03	-.12006E+03	-.12124E+03	-.13809E+03	-.12735E+03	-.12646E+03	-.12884E+03	-.11398E+03	-.12676E+03
104	-.20000E+03	-.20000E+03	-.12164E+03	-.11836E+03	-.12595E+03	-.12501E+03	-.11906E+03	-.12037E+03	-.11644E+03	-.12907E+03
105	-.44347E+02	-.41337E+02	-.40647E+02	-.11922E+03	-.11798E+03	-.12143E+03	-.11857E+03	-.11935E+03	-.12814E+03	-.11699E+03
106	-.12197E+03	-.11939E+03	-.12563E+03	-.13182E+03	-.12728E+03	-.12233E+03	-.12394E+03	-.12713E+03	-.11881E+03	-.11911E+03
107	-.44510E+02	-.11614E+03	-.13117E+03	-.12298E+03	-.12871E+03	-.11770E+03	-.11922E+03	-.13776E+03	-.11046E+03	-.12709E+03
108	-.20000E+03	-.11969E+03	-.14390E+03	-.12966E+03	-.12061E+03	-.12068E+03	-.11852E+03	-.11516E+03	-.11455E+03	-.12360E+03
109	-.44671E+02	-.41660E+02	-.12614E+03	-.12384E+03	-.12180E+03	-.12865E+03	-.11560E+03	-.11132E+03	-.12039E+03	-.12226E+03
110	-.11490E+03	-.14113E+03	-.12282E+03	-.13774E+03	-.12119E+03	-.12653E+03	-.11748E+03	-.12982E+03	-.12829E+03	-.12198E+03
111	-.44828E+02	-.13288E+03	-.12611E+03	-.11722E+03	-.13804E+03	-.12856E+03	-.11707E+03	-.13597E+03	-.11547E+03	-.15093E+03
112	-.20000E+03	-.20000E+03	-.20000E+03	-.11628E+03	-.11691E+03	-.11612E+03	-.12236E+03	-.12549E+03	-.12113E+03	-.13066E+03
113	-.44985E+02	-.41974E+02	-.41287E+02	-.41117E+02	-.12483E+03	-.12982E+03	-.12457E+03	-.11235E+03	-.11389E+03	-.12955E+03
114	-.11627E+03	-.12444E+03	-.12000E+03	-.15601E+03	-.13018E+03	-.13006E+03	-.12591E+03	-.13612E+03	-.12671E+03	-.12355E+03
115	-.45135E+02	-.11907E+03	-.11871E+03	-.12346E+03	-.15576E+03	-.12019E+03	-.13736E+03	-.11844E+03	-.11905E+03	-.13331E+03
116	-.20000E+03	-.12482E+03	-.11693E+03	-.12750E+03	-.11696E+03	-.13221E+03	-.14003E+03	-.12371E+03	-.12943E+03	-.12096E+03
117	-.45287E+02	-.42275E+02	-.12236E+03	-.12873E+03	-.12949E+03	-.11969E+03	-.12065E+03	-.11563E+03	-.11715E+03	-.12708E+03
118	-.11778E+03	-.13205E+03	-.13453E+03	-.11968E+03	-.12458E+03	-.11871E+03	-.12114E+03	-.13214E+03	-.13565E+03	-.11585E+03
119	-.45432E+02	-.13611E+03	-.11786E+03	-.13049E+03	-.14047E+03	-.12602E+03	-.11655E+03	-.12553E+03	-.11836E+03	-.12320E+03
120	-.20000E+03	-.20000E+03	-.11802E+03	-.13566E+03	-.12602E+03	-.12024E+03	-.12390E+03	-.12450E+03	-.12896E+03	-.12558E+03

K	$P_K(1-B)$	$P_K(2-B)$	$P_K(3-B)$	$P_K(4-B)$	$P_K(5-B)$	$P_K(6-B)$	$P_K(7-B)$	$P_K(8-B)$	$P_K(9-B)$	$P_K(10-B)$
121	-.45579E+02	-.42568E+02	-.41880E+02	-.12947E+03	-.11848E+03	-.11960E+03	-.13636E+03	-.12582E+03	-.11258E+03	-.15576E+03
122	-.11947E+03	-.11884E+03	-.13209E+03	-.12474E+03	-.12408E+03	-.12996E+03	-.12133E+03	-.13049E+03	-.12813E+03	-.11365E+03
123	-.45720E+02	-.12278E+03	-.12830E+03	-.12511E+03	-.12146E+03	-.12504E+03	-.12265E+03	-.11423E+03	-.13515E+03	-.12728E+03
124	-.20000E+03	-.11693E+03	-.13572E+03	-.13683E+03	-.13766E+03	-.13165E+03	-.12681E+03	-.13151E+03	-.11837E+03	-.12221E+03
125	-.45861E+02	-.42850E+02	-.13371E+03	-.12553E+03	-.12439E+03	-.12924E+03	-.12112E+03	-.11639E+03	-.10943E+03	-.12032E+03
126	-.12142E+03	-.12830E+03	-.12675E+03	-.13205E+03	-.13417E+03	-.12656E+03	-.13018E+03	-.12561E+03	-.12206E+03	-.12422E+03
127	-.45998E+02	-.14047E+03	-.13179E+03	-.12955E+03	-.13082E+03	-.13054E+03	-.13120E+03	-.12656E+03	-.12452E+03	-.12183E+03
128	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12660E+03	-.11217E+03	-.12041E+03	-.11502E+03
129	-.46135E+02	-.43125E+02	-.42437E+02	-.42268E+02	-.42227E+02	-.42216E+02	-.42212E+02	-.12266E+03	-.14831E+03	-.13981E+03
130	-.12047E+03	-.11920E+03	-.11690E+03	-.11762E+03	-.11705E+03	-.12059E+03	-.12946E+03	-.12075E+03	-.10562E+03	-.10919E+03
131	-.46267E+02	-.12579E+03	-.11543E+03	-.11666E+03	-.12093E+03	-.12239E+03	-.12002E+03	-.12100E+03	-.11787E+03	-.12487E+03
132	-.20000E+03	-.11885E+03	-.11708E+03	-.11689E+03	-.12335E+03	-.12037E+03	-.13056E+03	-.12952E+03	-.12955E+03	-.13078E+03
133	-.46400E+02	-.43388E+02	-.11952E+03	-.13598E+03	-.12484E+03	-.12617E+03	-.13358E+03	-.12123E+03	-.11807E+03	-.14400E+03
134	-.12250E+03	-.13684E+03	-.13071E+03	-.14170E+03	-.11765E+03	-.12250E+03	-.13566E+03	-.13148E+03	-.11946E+03	-.12142E+03
135	-.46528E+02	-.12281E+03	-.13064E+03	-.13530E+03	-.12056E+03	-.12057E+03	-.12164E+03	-.12337E+03	-.13717E+03	-.12175E+03
136	-.20000E+03	-.20000E+03	-.12580E+03	-.11571E+03	-.11906E+03	-.12211E+03	-.13197E+03	-.11581E+03	-.11596E+03	-.12225E+03
137	-.46657E+02	-.43647E+02	-.42958E+02	-.12254E+03	-.13053E+03	-.12229E+03	-.12736E+03	-.11535E+03	-.11806E+03	-.11766E+03
138	-.11544E+03	-.12302E+03	-.13763E+03	-.13607E+03	-.12576E+03	-.12383E+03	-.13871E+03	-.13053E+03	-.12702E+03	-.11941E+03
139	-.46781E+02	-.11872E+03	-.12326E+03	-.12014E+03	-.12044E+03	-.11932E+03	-.11397E+03	-.11726E+03	-.10422E+03	-.13729E+03
140	-.20000E+03	-.12247E+03	-.14397E+03	-.12176E+03	-.12422E+03	-.13372E+03	-.11680E+03	-.12131E+03	-.12886E+03	-.14943E+03
141	-.46908E+02	-.43895E+02	-.12921E+03	-.12621E+03	-.11825E+03	-.12283E+03	-.13784E+03	-.13865E+03	-.14664E+03	-.13565E+03
142	-.11658E+03	-.13730E+03	-.12410E+03	-.12082E+03	-.12341E+03	-.13025E+03	-.12069E+03	-.11503E+03	-.11856E+03	-.10906E+03
143	-.47028E+02	-.13540E+03	-.12801E+03	-.12105E+03	-.11942E+03	-.12764E+03	-.12665E+03	-.10951E+03	-.11986E+03	-.12925E+03
144	-.20000E+03	-.20000E+03	-.20000E+03	-.11961E+03	-.13870E+03	-.11920E+03	-.15028E+03	-.12512E+03	-.11433E+03	-.11176E+03
145	-.47151E+02	-.44140E+02	-.43452E+02	-.43287E+02	-.12678E+03	-.12595E+03	-.12663E+03	-.11618E+03	-.12203E+03	-.13629E+03
146	-.11781E+03	-.12411E+03	-.12085E+03	-.12638E+03	-.12920E+03	-.13865E+03	-.13893E+03	-.12595E+03	-.12437E+03	-.13991E+03
147	-.47269E+02	-.12159E+03	-.11813E+03	-.12278E+03	-.12179E+03	-.15516E+03	-.11853E+03	-.12657E+03	-.13942E+03	-.12109E+03
148	-.20000E+03	-.11674E+03	-.11862E+03	-.12494E+03	-.12581E+03	-.12344E+03	-.12824E+03	-.13788E+03	-.13219E+03	-.11972E+03
149	-.47387E+02	-.44375E+02	-.12563E+03	-.11946E+03	-.11554E+03	-.11308E+03	-.13256E+03	-.10892E+03	-.97789E+02	-.12073E+03
150	-.11916E+03	-.12825E+03	-.14736E+03	-.11977E+03	-.12246E+03	-.12292E+03	-.12227E+03	-.12423E+03	-.11561E+03	-.10702E+03
151	-.47501E+02	-.13866E+03	-.12153E+03	-.12739E+03	-.13297E+03	-.14180E+03	-.11895E+03	-.12262E+03	-.12663E+03	-.12677E+03
152	-.20000E+03	-.20000E+03	-.12107E+03	-.13145E+03	-.11786E+03	-.11560E+03	-.12462E+03	-.11128E+03	-.14695E+03	-.13402E+03
153	-.47617E+02	-.44606E+02	-.43918E+02	-.13350E+03	-.11857E+03	-.12548E+03	-.12881E+03	-.12434E+03	-.11813E+03	-.12422E+03
154	-.11858E+03	-.12129E+03	-.12508E+03	-.11969E+03	-.13091E+03	-.12629E+03	-.12566E+03	-.13049E+03	-.11504E+03	-.12444E+03
155	-.47728E+02	-.12378E+03	-.13322E+03	-.11994E+03	-.14709E+03	-.11959E+03	-.12051E+03	-.12159E+03	-.12937E+03	-.11775E+03
156	-.20000E+03	-.11851E+03	-.17028E+03	-.14546E+03	-.12937E+03	-.12522E+03	-.12176E+03	-.11360E+03	-.11744E+03	-.11353E+03
157	-.47842E+02	-.44830E+02	-.13645E+03	-.14732E+03	-.13900E+03	-.13001E+03	-.12123E+03	-.11303E+03	-.12821E+03	-.12171E+03
158	-.11996E+03	-.13470E+03	-.12727E+03	-.12572E+03	-.13213E+03	-.11236E+03	-.12304E+03	-.12519E+03	-.11414E+03	-.12928E+03
159	-.47949E+02	-.14127E+03	-.13323E+03	-.12860E+03	-.11935E+03	-.12027E+03	-.12160E+03	-.14980E+03	-.12931E+03	-.12204E+03
160	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.11792E+03	-.11605E+03	-.13517E+03	-.13390E+03	-.12248E+03	-.12009E+03
161	-.48060E+02	-.45049E+02	-.44362E+02	-.44193E+02	-.44150E+02	-.12918E+03	-.11675E+03	-.12415E+03	-.11364E+03	-.11284E+03
162	-.12152E+03	-.12217E+03	-.11841E+03	-.11850E+03	-.13286E+03	-.13305E+03	-.12774E+03	-.11746E+03	-.12315E+03	-.12976E+03
163	-.48165E+02	-.11848E+03	-.11661E+03	-.11652E+03	-.12917E+03	-.12027E+03	-.13351E+03	-.12525E+03	-.11547E+03	-.12437E+03
164	-.13251E+03	-.12113E+03	-.12836E+03	-.12193E+03	-.12349E+03	-.12425E+03	-.12336E+03	-.13090E+03	-.13177E+03	-.12565E+03
165	-.48273E+02	-.45261E+02	-.12215E+03	-.14775E+03	-.14019E+03	-.11746E+03	-.12058E+03	-.12298E+03	-.11537E+03	-.11877E+03
166	-.12076E+03	-.12557E+03	-.12392E+03	-.12882E+03	-.13431E+03	-.11027E+03	-.11402E+03	-.12334E+03	-.12224E+03	-.12836E+03
167	-.48376E+02	-.13495E+03	-.14151E+03	-.13237E+03	-.13029E+03	-.11723E+03	-.12084E+03	-.11125E+03	-.12669E+03	-.11965E+03
168	-.13272E+03	-.20000E+03	-.11811E+03	-.12194E+03	-.15438E+03	-.12268E+03	-.11478E+03	-.12298E+03	-.11984E+03	-.12165E+03
169	-.48481E+02	-.45471E+02	-.44781E+02	-.12535E+03	-.12133E+03	-.12560E+03	-.12884E+03	-.13975E+03	-.11177E+03	-.11312E+03
170	-.12237E+03	-.11870E+03	-.12292E+03	-.13274E+03	-.12111E+03	-.13015E+03	-.16153E+03	-.11233E+03	-.10532E+03	-.13711E+03
171	-.48582E+02	-.12083E+03	-.12365E+03	-.12295E+03	-.11991E+03	-.12696E+03	-.13224E+03	-.10441E+03	-.11792E+03	-.11798E+03
172	-.13292E+03	-.12482E+03	-.12858E+03	-.13130E+03	-.12916E+03	-.11365E+03	-.11238E+03	-.12745E+03	-.12215E+03	-.11066E+03
173	-.48685E+02	-.45672E+02	-.13194E+03	-.12253E+03	-.12100E+03	-.11658E+03	-.12409E+03	-.12189E+03	-.11056E+03	-.98727E+02
174	-.11644E+03	-.12597E+03	-.13628E+03	-.12138E+03	-.12866E+03	-.12185E+03	-.11333E+03	-.13252E+03	-.14036E+03	-.12483E+03
175	-.48788E+02	-.13753E+03	-.12772E+03	-.12714E+03	-.12520E+03	-.12913E+03	-.13299E+03	-.12112E+03	-.11589E+03	-.12032E+03
176	-.13312E+03	-.20000E+03	-.20000E+03	-.12267E+03	-.12847E+03	-.12661E+03	-.11345E+03	-.11443E+03	-.12302E+03	-.12196E+03
177	-.48893E+02	-.45872E+02	-.45184E+02	-.45015E+02	-.13034E+03	-.12663E+03	-.11746E+03	-.11941E+03	-.13347E+03	-.12279E+03
178	-.11742E+03	-.11778E+03	-.11917E+03	-.12173E+03	-.12607E+03	-.12603E+03	-.12046E+03	-.11611E+03	-.12501E+03	-.12591E+03
179	-.48978E+02	-.12371E+03	-.11900E+03	-.15175E+03	-.12511E+03	-.14224E+03	-.13494E+03	-.12270E+03	-.13552E+03	-.15160E+03
180	-.20000E+03	-.11794E+03	-.12555E+03	-.12239E+03	-.11874E+03	-.13173E+03	-.15527E+03	-.15093E+03	-.12542E+03	-.12946E+03
181	-.49077E+02	-.46065E+02	-.12785E+03	-.13084E+03	-.14831E+03	-.12107E+03	-.13913E+03	-.13009E+03	-.10385E+03	-.13033E+03
182	-.11848E+03	-.17357E+03	-.12096E+03	-.12481E+03	-.14690E+03	-.12511E+03	-.12896E+03	-.12644E+03	-.13540E+03	-.11990E+03
183	-.49170E+02	-.13948E+03	-.12384E+03	-.13459E+03	-.12755E+03	-.12068E+03	-.12145E+03	-.10543E+03	-.11762E+03	-.11689E+03
184	-.20000E+03	-.20000E+03	-.12317E+03	-.14062E+03	-.14605E+03	-.12940E+03	-.13104E+03	-.12325E+03	-.12081E+03	-.11847E+03
185	-.49267E+02	-.46256E+02	-.45568E+02	-.12642E+03	-.12786E+03	-.12807E+03	-.12201E+03	-.12003E+03	-.11917E+03	-.13048E+03
186	-.11963E+03	-.11963E+03	-.14546E+03	-.12024E+03	-.12051E+03	-.13086E+03	-.11320E+03	-.12973E+03	-.11712E+03	-.13114E+03
187	-.49358E+02	-.12592E+03	-.16106E+03	-.12153E+03	-.12634E+03	-.12960E+03	-.11487E+03	-.13133E+03	-.11497E+03	-.13316E+03
188	-.20000E+03	-.12024E+03	-.13640E+03	-.12552E+03	-.12510E+03	-.12180E+03	-.12318E+03	-.12404E+03	-.10202E+03	-.10872E+03
189	-.49455E+02	-.46441E+02	-.12906E+03	-.13705E+03	-.12861E+03	-.12826E+03	-.11793E+03	-.12717E+03	-.12636E+03	-.12033E+03
190	-.12089E+03	-.18062E+03	-.13343E+03	-.13005E+03	-.12896E+03	-.12173E+03	-.11676E+03	-.12971E+03	-.12748E+03	-.12271E+03
191	-.49542E+02	-.14350E+03	-.13274E+03	-.12898E+03	-.13475E+03	-.13245E+03	-.13048E+03	-.13637E+03	-.11741E+03	-.12945E+03
192	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.20000E+03	-.12107E+03	-.13398E+03	-.12188E+03	-.10913E+03	-.11909E+03
193	-.49635E+02	-.46624E+02	-.45936E+02	-.45768E+02	-.45726E+02	-.45715E+02	-.13052E+03	-.11580E+03	-.11706E+03	-.12268E+03
194	-.12231E+03	-.12229E+03	-.11866E+03	-.11851E+03	-.11818E+03	-.12870E+03	-.12217E+03	-.12408E+03	-.13911E+03	-.12604E+03
195	-.49722E+02	-.12029E+03	-.11814E+03	-.11768E+03	-.12722E+03	-.12383E+03	-.11622E+03	-.11270E+03	-.12209E+03	-.14727E+03
196	-.20000E+03	-.12308E+03	-.11803E+03	-.12039E+03	-.12921E+03	-.14329E+03	-.11832E+03	-.11222E+03	-.12566E+03	-.12491E+03
197	-.49813E+02	-.46800E+02	-.12447E+03	-.14203E+03	-.12153E+03	-.12527E+03	-.11821E+03	-.11483E+03	-.11124E+03	-.11692E+03
198	-.12392E+03	-.18664E+03	-.14481E+03	-.13406E+03	-.12453E+03	-.14326E+03	-.12706E+03	-.12700E+03	-.12789E+03	-.13350E+03
199	-.49899E+02	-.13676E+03	-.15338E+03	-.13685E+03	-.12033E+03	-.12939E+03	-.11496E+03	-.12017E+03	-.11384E+03	-.12257E+03
200	-.20000E+03	-.20000E+03	-.12033E+03	-.12621E+03	-.11660E+03	-.13579E+03	-.10965E+03	-.14356E+03	-.11938E+03	-.12281E+03

APPENDIX D. SIMULATION SOFTWARE AND RESULTS

A. SIMULATION SOFTWARE

The program is listed in Figure 34.

B. SIMULATION RESULTS

Figure 35 to Figure 40 show the various spectral plots. Table 8 is the summary of the simulation results.

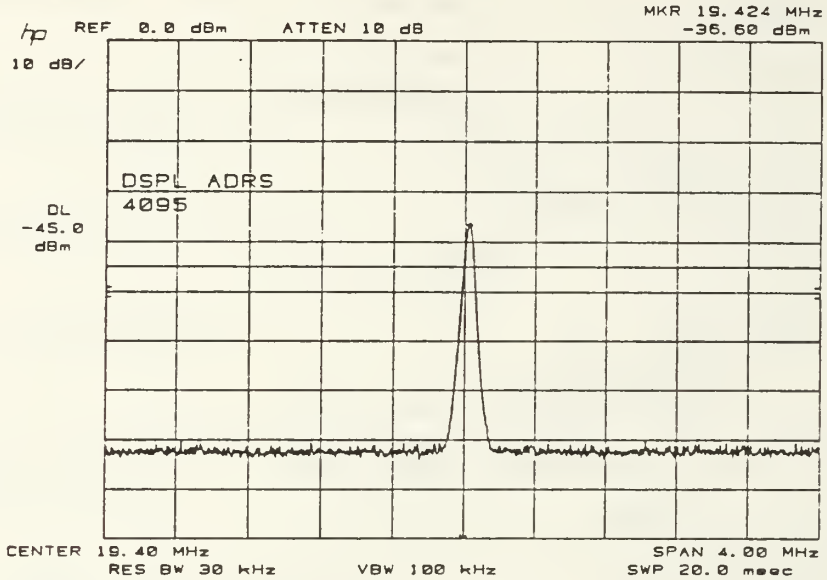
Table 8. SIMULATION RESULTS

Bit	P_0 (dB)	P_{-1} (dB)	P_{-1} (dB)	T.L. (dB)	S.R. (dB)
1	-36.6	-41.6	-41.6	5.0	0.0
2	-36.6	-38.6	-47.5	2.0	8.9
3	36.6	-36.7	-53	0.1	16.3
4	-36.6	-36.6	-60	0.0	23.4
5	-36.6	-36.6	-65	0.0	29.4
6	-36.6	-36.6	-69	0.0	32.4
7	-36.6	-36.6	-72	0.0	35.4
8	-36.6	-36.6	-74	0.0	37.4
9	-36.6	-36.6	-	0.0	-
10	-36.6	-36.6	-	0.0	-

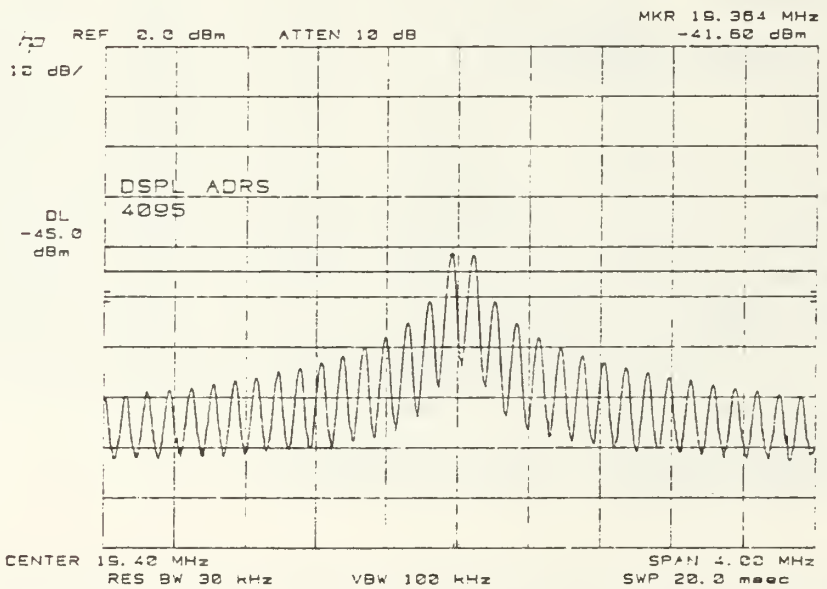
```
DEFINE WGDPS
TDOMAIN
(****************** SET UP PARAMETER ***************)
"HOW MANY BIT ? 0 to 10 BITS, PLEASE." ? CR
ENTER ? STORE BIT CR
"HOW MANY ELEMENT ?" ? CR
"IN POWER OF 2 (e.g 2,4,8,16...), PLEASE." ? CR
ENTER ? STORE ELEM CR
(*************** HELP USER TO GET FSET ************)
"WHAT IS THE CARRIER FREQUENCY ?" ? CR
"IN MHz, PLEASE." ? CR
ENTER ? STORE FCTEMP CR
FCTEMP ELEM * 125 / STORE FSETTEMP
"FSET SHOULD BE AROUND "? FSETTEMP ? CR
"YOU DECIDE WHAT THE FSET IS."? CR
ENTER ? STORE FSET CR
(************ COMPUTE FC, FM AND FCPRIME **********)
FSET ELEM ? 125 * STORE FC
125 ELEM / STORE FM
FC FM + STORE FCPRIME
"FC      =" ? FC ? "MHz" ? CR
"FM      =" ? FM ? "MHz" ? CR
"FCPRIME =" ? FCPRIME ? "MHz" ? CR
(***************** COMPUTE THE CONTEXT *************)
ELEM CTX 0 LOAD
(********* SET UP THE PHASE OF BASIC CARRIER ******)
RAMP PI FSET * * STORE A
(********** SET UP THE PHASE OF MODULATION ********)
2 BIT POW STORE SUBELEM
"SUBELEMENT ="? SUBELEM ? CR
ELEM SUBELEM ? STORE MDIV
"MDIV ="? MDIV ? CR
2 BIT 1 - POW STORE DIVSTEP
"DIVSTEP ="? DIVSTEP ? CR
SUBELEM STORE LOOP
"LOOP ="? LOOP ? CR
I=0 TO LOOP REPEAT
(MDIV I *              STORE START ?
 MDIV I 1 + * 1 -      STORE STOPP ?
 PI I 1 + * DIVSTEP / STORE STEPP ? CR
 STOPP ELEM GE IFTRUE GOTO PLOTT
 START STOPP WI
 STEPP LOAD                          )
GOTO PLOTT
END

DEFINE PLOTT
(************** SUM BOTH PHASE COMPONENT ***********)
FULL STORE B
A B + STORE C
(********** DISPLAY ALL FOR CHECKING **************)
ELEM CTX ? 0 LOAD ?
B ? 5 WAIT A ? 5 WAIT C ? 5 WAIT
(***************** GET THE COS WAVEFORM ************)
COS ?
(********* SET LOAD ATTENUATION 50 dB *************)
"50" $ATTENUATE
(*************** DOWNLOAD TO AWS AND GO ***********)
"WAVELOAD: AUTO" $DOWNLOAD
GO
END
```

Figure 34. Listing of simulation software

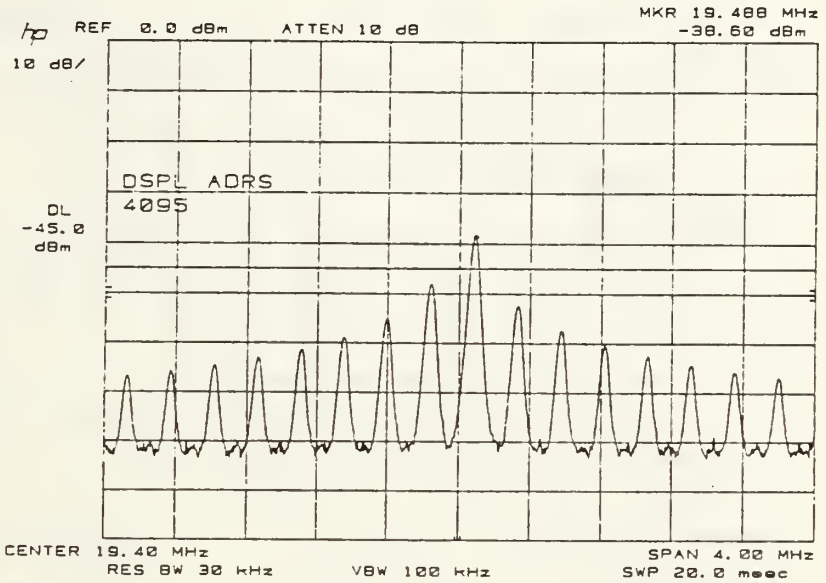


(a) unserrodyned case

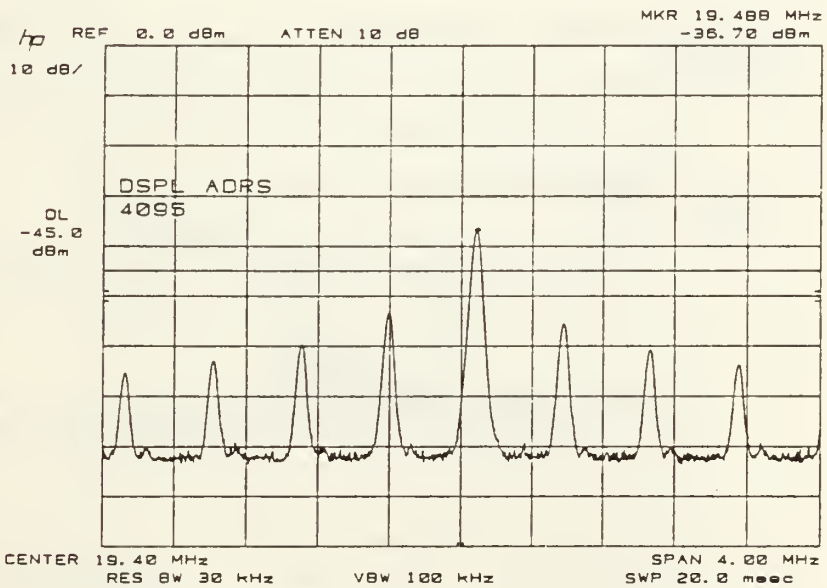


(b) 1-Bit serrodyne

Figure 35. Spectral plots for unserrodyned and 1-bit serrodyne signals

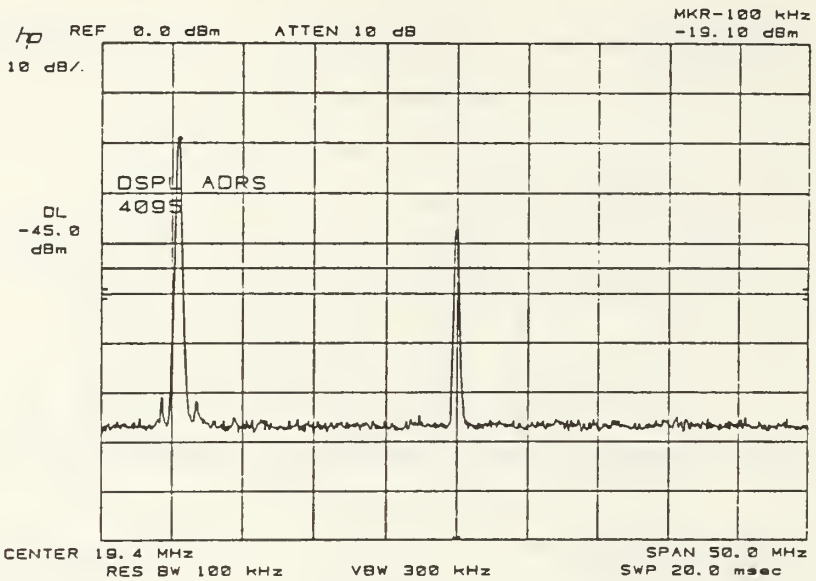


(a) 2-Bit serrodyne

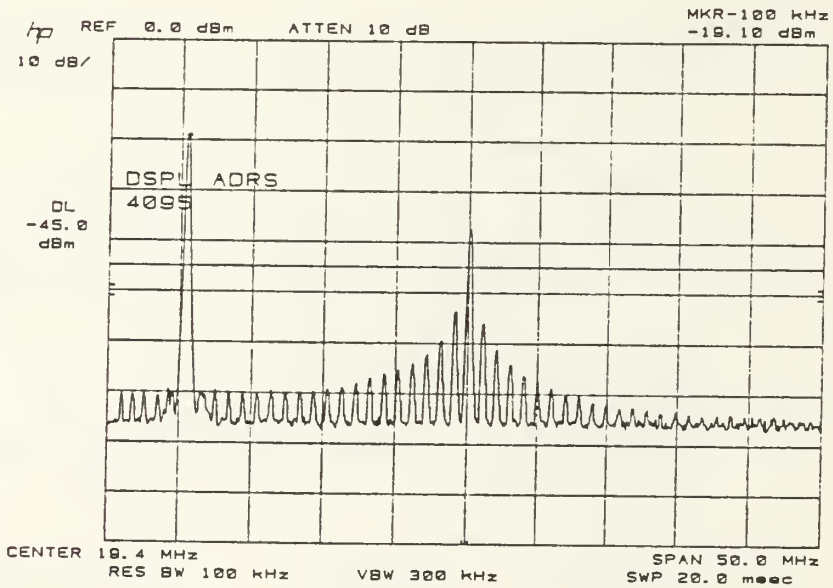


(b) 3-Bit serrodyne

Figure 36. Spectral plots for 2-bit and 3-bit serrodyne signals

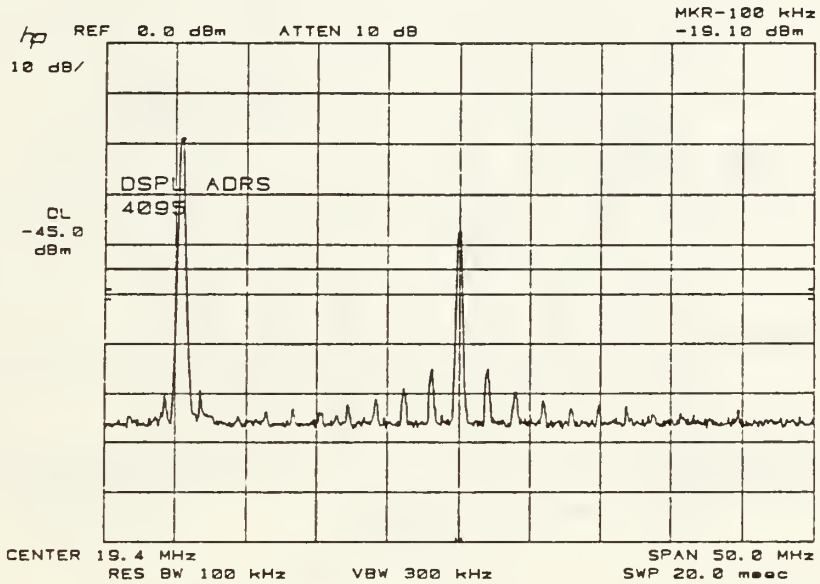


(a) unserrodyned case

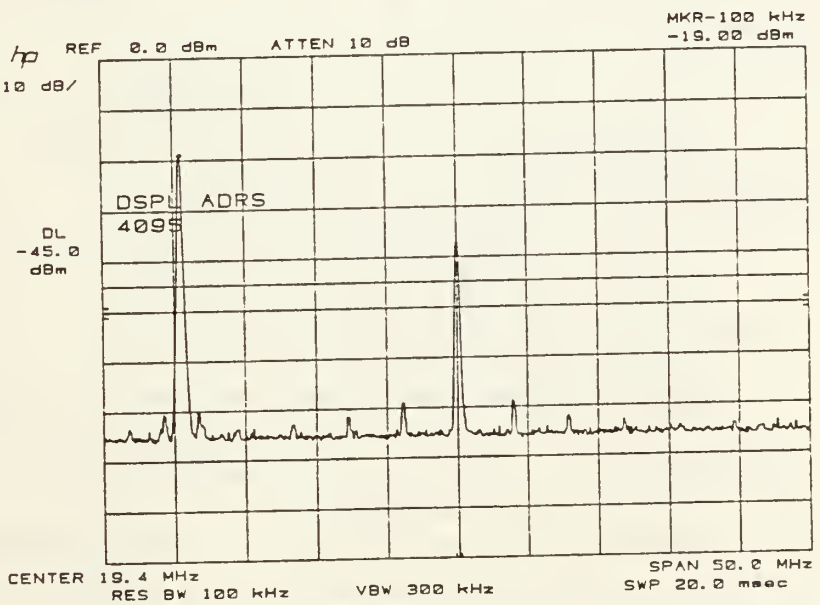


(b) 4-Bit serrodyne

Figure 37. Spectral plots for unserrodyned and 4-bit serrodyne signals: The frequency span used here is 50 MHz instead of 4 MHz as in Figure 35 on page 74.

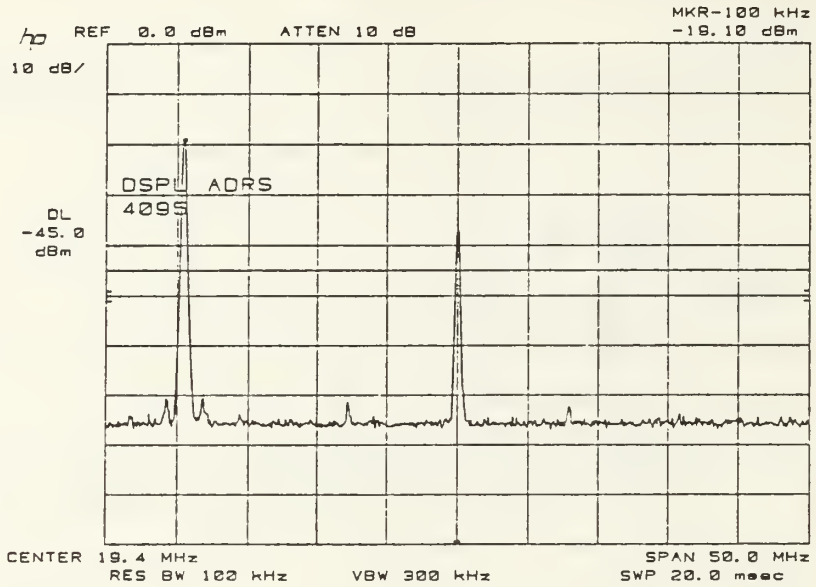


(a) 5-Bit serrodyne

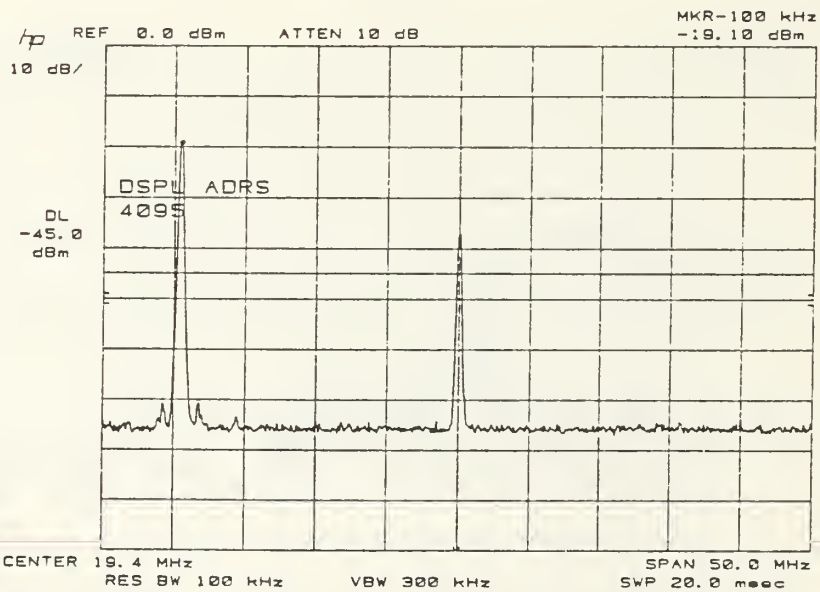


(b) 6-Bit serrodyne

Figure 38. Spectral plots for 5-bit and 6-bit serrodyne signals

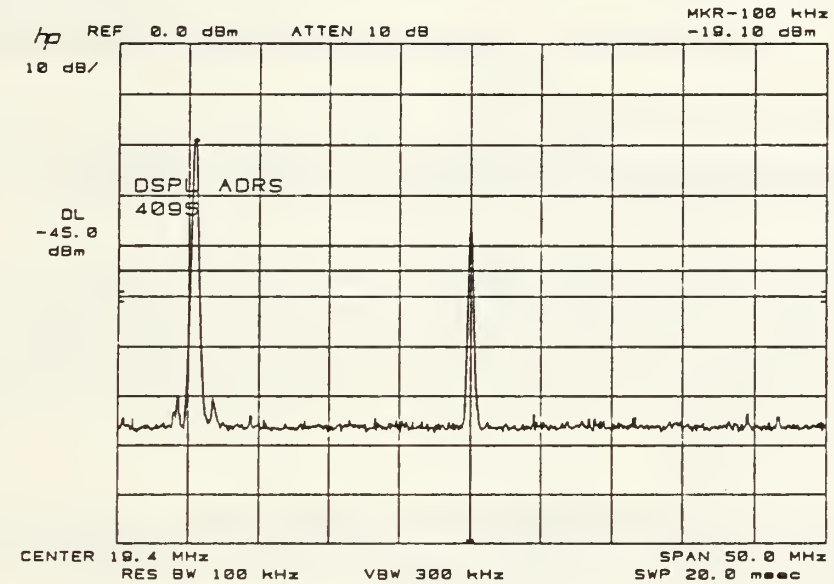


(a) 7-Bit serrodyne

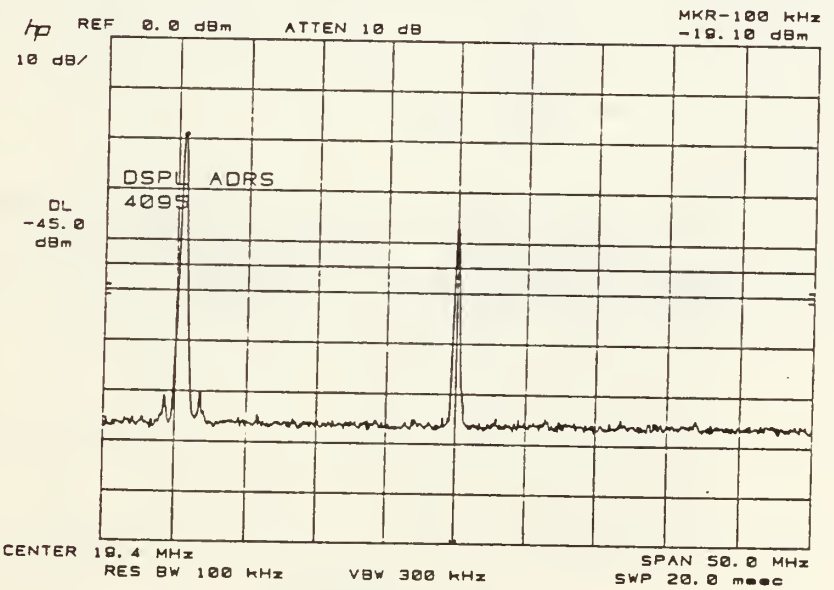


(b) 8-Bit serrodyne

Figure 39. Spectral plots for 7-bit and 8-bit serrodyne signals



(a) 9-Bit serrodyne



(b) 10-Bit serrodyne

Figure 40. Spectral plots for 9-bit and 10-bit serrodyne signals: The difference between the noise floor and the peak carrier power is only about 50 dB in our case. Therefore , the S.R. cannot be measured for $B \geq 9$ bits.

APPENDIX E. LABORATORY MEASURED DATA

A. PIN DESCRIPTION AND CONTROL CIRCUIT FOR THE DEVICE

The pin description of the DPS is as shown in Table 9 and the control circuit is shown in Figure 41. With this control circuit, the bit control is tabulated in Table 10. The input frequency of the clock will always be $64 \cdot f_T$ (since $2^6 = 64$) to give the translation frequency of f_T.

Table 9. PIN DESCRIPTION OF THE DPS

Pin number	Description
1	5.6^o (LSB)
2	11.25^o
3	22.5^o
4	45^o
5	90^o
6	180^o (MSB)
7	+ 5 Volts
8	-5 Volts
9	Ground
Note : The control signals for pin 1 to 6 are active low.	

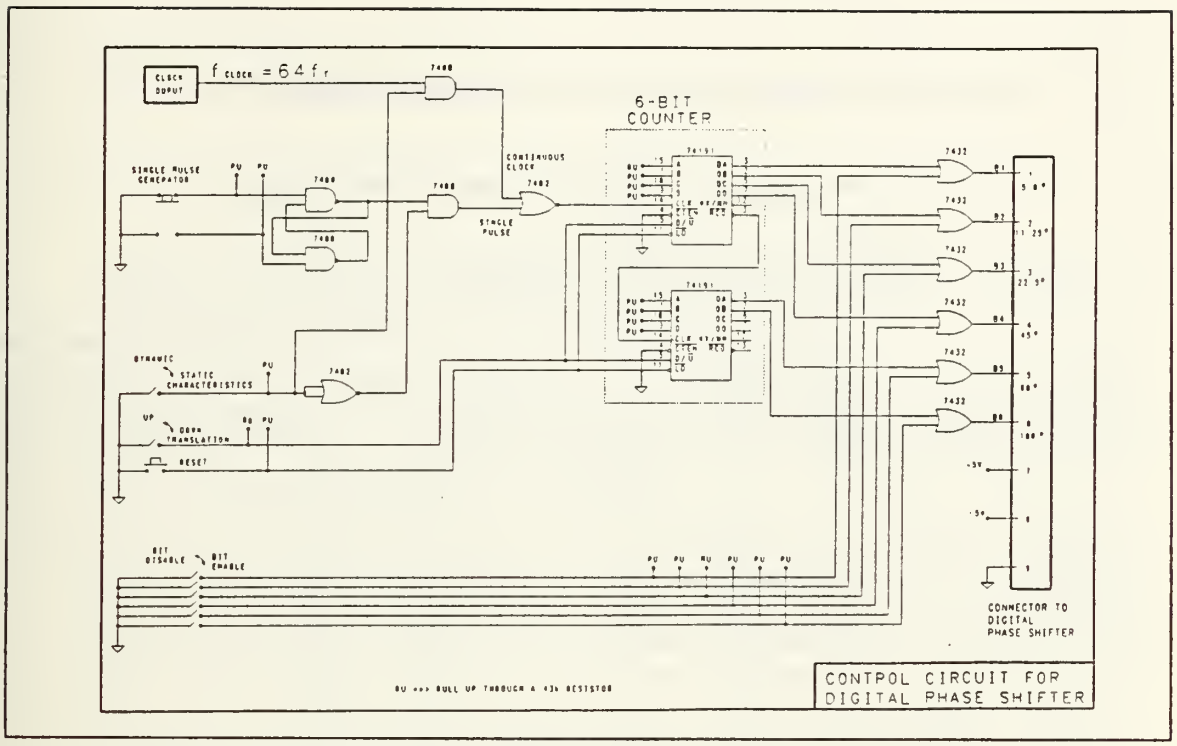


Figure 41. Control circuit for the digital phase shifter

Table 10. CONTROL OF BITS

f_{clock}	f_T	Number of bit employed	Control of bits					
			B6	B5	B4	B3	B2	B1
$64 \cdot f_T$	f_T	6	E	E	E	E	E	E
		5	E	E	E	E	E	D
		4	E	E	E	E	D	D
		3	E	E	E	D	D	D
		2	E	E	D	D	D	D
		1	E	D	D	D	D	D
E:Enabled D:Disabled								

B. LABORATORY DATA FOR STATIC CHARACTERISTICS

The insertion loss and phase shift were recorded (Table 11).

Table 11. S-PARAMETER MEASUREMENT: S_{21} for $f_0 = 10.25$ GHz.

State	Deg	S_{21}		ΔAng	•	State	Deg	S_{21}		ΔAng
		dB	Ang					dB	Ang	
111111	0	8.75	52.4	0	•	011111	180	7.95	-124.8	182.8
111110	5.625	8.47	59.3	6.9	•	011110	185.625	7.69	-119.6	188.0
111101	11.25	8.55	65.6	13.1	•	011101	191.25	7.77	-112.8	194.8
111100	16.875	8.35	71.1	18.7	•	011100	196.875	7.50	-107.4	200.2
111011	22.5	8.55	72.1	19.7	•	011011	202.5	7.79	-106.2	201.4
111010	28.125	8.34	76.5	24.1	•	011010	208.125	7.47	-100.7	206.9
111001	33.75	8.45	84.2	31.8	•	011001	213.75	7.65	-94.0	213.6
111000	39.375	8.20	89.7	37.3	•	011000	219.375	7.35	-88.4	219.2
110111	45	8.90	99.6	47.2	•	010111	225	8.05	-79.7	227.9
110110	50.625	8.40	104.1	51.7	•	010110	230.625	7.55	-74.7	232.9
110101	56.25	8.67	112.6	60.2	•	010101	236.25	7.80	-67.1	240.5
110100	61.876	8.17	117.2	64.8	•	010100	241.875	7.30	-62.2	245.4
110011	67.5	8.70	116.8	64.4	•	010011	247.5	7.75	-61.3	246.3
110010	73.125	8.20	121.9	69.5	•	010010	253.125	7.28	-56.0	251.6
110001	78.75	8.55	138.6	86.2	•	010001	258.75	7.60	-48.5	259.1
110000	84.375	8.05	135.3	82.9	•	010000	264.375	7.15	-43.3	264.3
101111	90	7.98	138.6	86.2	•	001111	270	7.22	-38.2	269.4
101110	95.625	7.55	144.8	92.4	•	001110	275.625	6.85	-31.9	275.7
101101	101.25	7.87	151.6	99.2	•	001101	281.25	7.15	-26.0	281.6
101100	106.875	7.45	157.9	105.5	•	001100	286.875	6.80	-19.9	287.7
101011	112.5	7.90	160.0	107.6	•	001011	292.5	7.30	-17.2	290.4
101010	118.125	7.55	166.5	114.1	•	001010	298.125	7.00	-11.1	296.5
101001	123.75	7.84	172.9	120.5	•	001001	303.75	7.25	-5.4	302.2
101000	129.375	7.50	179.3	126.9	•	001000	309.375	6.95	0.6	308.2
100111	135	7.65	-177.3	130.3	•	000111	315	6.90	6.1	313.7
100110	140.625	7.25	-171.2	136.4	•	000110	320.625	6.55	11.7	319.3
100101	146.25	7.60	-164.3	143.3	•	000101	326.25	6.95	18.2	325.8
100100	151.875	7.20	-158.1	149.5	•	000100	331.875	6.55	23.9	331.5
100011	157.5	7.60	-154.4	153.2	•	000011	337.5	7.12	28.3	335.9
100010	163.125	7.25	-148.9	158.7	•	000010	343.125	6.74	33.6	341.2
100001	168.75	7.55	-141.9	165.7	•	000001	348.75	7.12	39.9	347.5
100000	174.375	7.20	-136.4	171.2	•	000000	354.375	6.72	45.3	352.9

C. LABORATORY DATA FOR SERRODYNING EFFECT

a. Basic Characteristics of Serrodyning Effect

Based on the parameters given, a base set has been chosen for comparison purpose. It comprises the following parameters :

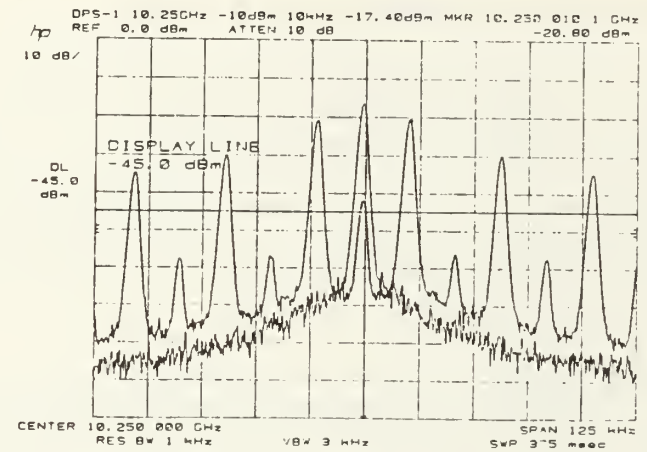
- Number B of of bits = 1, 2, 3, 4, 5 and 6
- Carrier frequency, f_{in} = 10.25 GHz (mid-frequency of the operating range)
- Input power, P_{in} = -10 dBm (measured at the output of the signal generator)
- Translation frequency, f_T = + 10 kHz

The spectral plots for the measurements for 1-bit to 6-bit are shown in Figure 42 to Figure 44. The unserrodyned spectrum has been obtained by re-setting all the control bits to inactive states (i.e., '1's in this case). Table 12 records the summary of the experimental data. From Table 12, it is also observed that T.L. is negative for the 3-, 4-, 5- and 6-Bit cases.

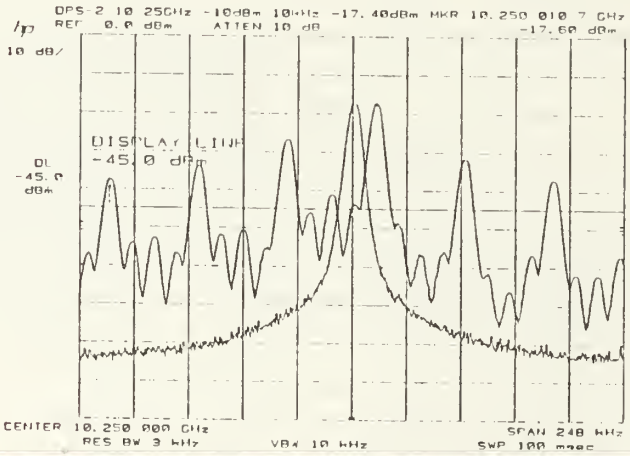
Table 12. EFFECT OF NUMBER OF BIT ON T.L. AND S.R.

f_{in} (GHz)	P_{in} (dBm)	f_T (kHz)	Bit	P_0 (dBm)	P_{-N-1} (dBm)	P_1 (dBm)	P^*_{N-1} (dBm)	*T.L.* (dB)	*S.R.* (dB)
10.25	-10	+ 10	1	-17.40	-20.80	-20.80	-30.20	3.40	0.00
			2	-17.40	-26.70	-17.60	-31.70	0.20	9.10
			3	-17.40	-32.20	-16.70	-36.60	-0.70	15.50
			4	-17.40	-36.90	-16.60	-43.00	-0.80	20.30
			5	-17.40	-37.30	-16.10	-41.00	-1.30	21.20
			6	-17.40	-39.50	-16.00	-38.10	-1.40	22.10

* = = or any other upper harmonic component with amplitude higher than P_{N-1}

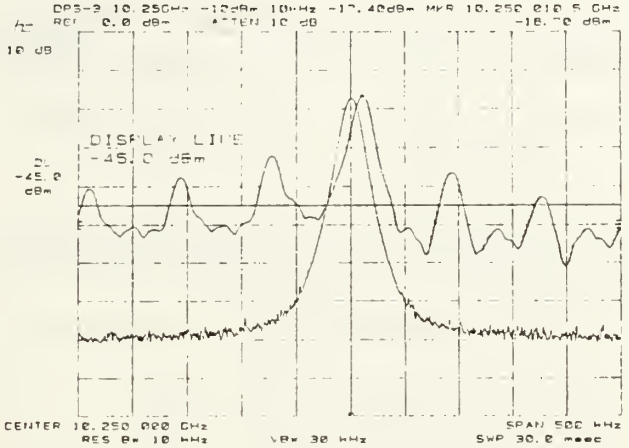


(a) 1-bit

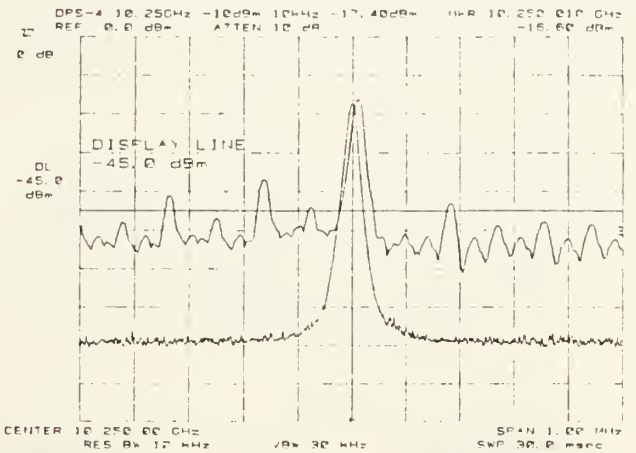


(b) 2-bit

Figure 42. Spectral plots for 1-bit and 2-bit serrodyne operation: The top and bottom traces are the spectrums of the serrodyne and unserrodyned signals respectively. The unserrodyned spectrum is obtained by resetting all the control bits to inactive states (i.e., '1's in this case). The frequency span is doubled from (a) to (b) so that each spectrum can contain at least four undesired side frequencies.

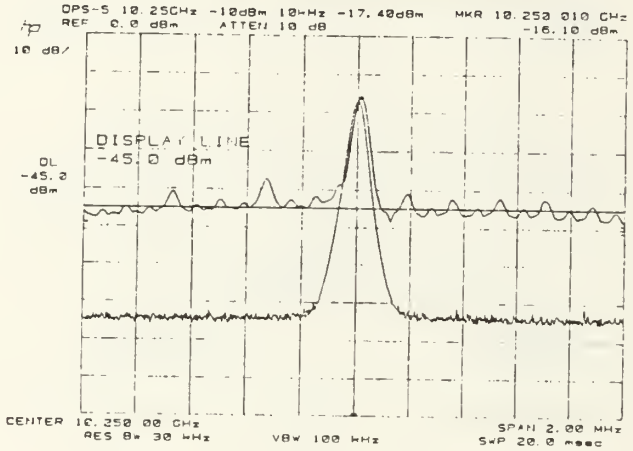


(a) 3-bit

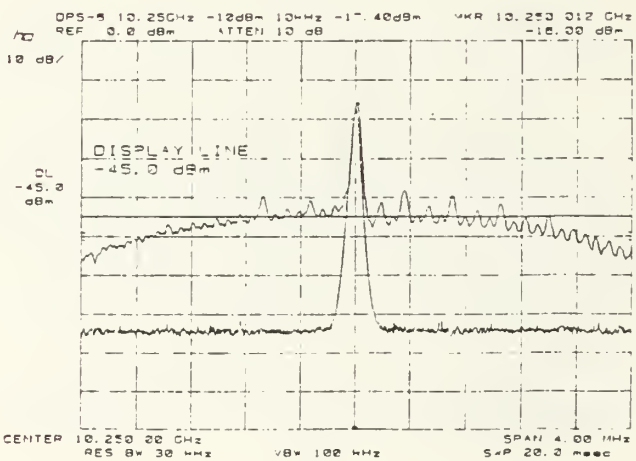


(b) 4-bit

Figure 43. Spectral plots for 3-bit and 4-bit serrodyne operation: The top and bottom traces are the spectrums of the serrodyne and unserrodyned signals respectively. The unserrodyned spectrum is obtained by resetting all the control bits to inactive states (i.e., '1's in this case). The frequency span is doubled from (a) to (b) so that each spectrum can contain at least four undesired side frequencies.



(a) 5-bit



(b) 6-bit

Figure 44. Spectral plots for 5-bit and 6-bit serrodyne operation: The top and bottom traces are the spectrums of the serrodyne and unserrodyned signals respectively. The unserrodyned spectrum is obtained by resetting all the control bits to inactive states (i.e., '1's in this case). The frequency span is doubled from (a) to (b) so that each spectrum can contain at least four undesired side frequencies.

b. Others

The following data have been recorded:

- Effect of down-translation frequency on T.L. and S.R. (Table 13)
- Effect of translation frequency on T.L. and S.R. (Table 14 and Table 15)
- Effect of carrier frequency on T.L. and S.R. (Table 16)
- Effect of carrier input power on T.L. and S.R. (Table 17)

Table 13. EFFECT OF DOWN-TRANSLATION FREQUENCY ON T.L. AND S.R.

f_{in} (GHz)	P_{in} (dBm)	f_T (kHz)	Bit	P_0 (dBm)	P_{-N-1} (dBm)	P_{-1} (dBm)	P^*_{N-1} (dBm)	*T.L.* (dB)	*S.R.* (dB)
10.25	-10	-10	1	-17.40	-30.20	-20.80	-20.80	3.40	0.00
			2	-17.40	-31.60	-17.60	-26.80	0.20	9.20
			3	-17.40	-36.80	-16.90	-32.80	-0.50	15.90
			4	-17.40	-43.00	-16.70	-37.10	-0.70	20.40
			5	-17.40	-44.50	-16.20	-38.10	-1.20	21.90
			6	-17.40	-40.70	-16.10	-38.60	-0.80	22.50
* = = > or any other upper harmonic component with amplitude higher than P_{N-1}									

Table 14. EFFECT OF TRANSLATION FREQUENCY ON T.L. AND S.R. (1- TO 3-BIT)

f_{in} (GHz)	P_{in} (dBm)	Bit	f_T (kHz)	P_0 (dBm)	P_{-N-1} (dBm)	P_1 (dBm)	P^*_{N-1} (dBm)	*T.L.* (dB)	*S.R.* (dB)
10.25	-10	1	0.5	-17.40	-20.90	-20.90	-30.30	3.5	0.00
10.25	-10	1	1	-17.40	-20.90	-20.90	-30.30	3.5	0.00
10.25	-10	1	2	-17.40	-20.60	-20.60	-30.00	3.20	0.00
10.25	-10	1	4	-17.20	-20.80	-20.80	-30.10	3.60	0.00
10.25	-10	1	6	-17.40	-20.70	-20.70	-30.10	3.30	0.00
10.25	-10	1	8	-17.40	-20.70	-20.70	-30.10	3.30	0.00
10.25	-10	1	10	-17.40	-20.80	-20.80	-30.20	3.40	0.00
10.25	-10	1	20	-17.40	-20.70	-20.70	-30.20	3.30	0.00
10.25	-10	1	40	-17.40	-20.80	-20.80	-30.30	3.40	0.00
10.25	-10	1	80	-17.40	-20.70	-20.70	-30.40	3.30	0.00
10.25	-10	1	160	-17.40	-21.00	-21.00	-32.20	3.60	0.00
10.25	-10	2	0.5	-17.40	-27.20	-17.60	-31.50	0.20	9.60
10.25	-10	2	1	-17.40	-27.10	-17.60	-31.50	0.20	9.50
10.25	-10	2	2	-17.20	-26.80	-17.40	-31.20	0.20	9.40
10.25	-10	2	4	-17.20	-26.60	-17.40	-31.20	0.20	9.20
10.25	-10	2	6	-17.40	-26.70	-17.50	-31.40	0.10	9.20
10.25	-10	2	8	-17.40	-26.30	-17.20	-31.20	-0.20	9.10
10.25	-10	2	10	-17.40	-26.70	-17.60	-31.70	0.20	9.10
10.25	-10	2	20	-17.40	-26.40	-17.60	-31.90	0.20	8.80
10.25	-10	2	40	-17.40	-25.90	-17.60	-32.80	0.20	8.30
10.25	-10	2	80	-17.40	-25.60	-18.00	-35.10	0.60	7.60
10.25	-10	2	160	-17.40	-25.50	-19.00	-33.60	1.60	6.50
10.25	-10	3	0.5	-17.40	-33.60	-16.90	-35.90	-0.50	16.70
10.25	-10	3	1	-17.40	-33.30	-16.80	-35.80	-0.60	16.50
10.25	-10	3	2	-17.20	-33.20	-16.70	-35.80	-0.50	16.50
10.25	-10	3	4	-17.20	-32.90	-16.70	-36.00	-0.50	16.20
10.25	-10	3	6	-17.40	-32.20	-16.40	-35.70	-1.00	15.80
10.25	-10	3	8	-17.40	-32.40	-16.60	-36.10	-0.80	15.80
10.25	-10	3	10	-17.40	-32.20	-16.70	-36.60	-0.70	15.50
10.25	-10	3	20	-17.40	-31.70	-17.00	-38.10	-0.40	14.70
10.25	-10	3	40	-17.40	-30.00	-17.20	-38.70	-0.20	12.80
10.25	-10	3	80	-17.40	-28.90	-17.80	-34.60	0.40	11.10
10.25	-10	3	160	-17.40	-29.30	-19.30	-31.20	1.90	10.00

* = = = or any other upper harmonic component with amplitude higher than P_{N-1}

Table 15. EFFECT OF TRANSLATION FREQUENCY ON T.L. AND S.R. (4- TO 6-BIT)

f_{in} (GHz)	P_{in} (dBm)	Bit	f_T (kHz)	P_0 (dBm)	P_{-N-1} (dBm)	P_1 (dBm)	P^*_{N-1} (dBm)	*T.L.* (dB)	*S.R.* (dB)
10.25	-10	4	0.5	-17.40	-39.90	-16.70	-41.30	-0.70	23.20
10.25	-10	4	1	-17.40	-39.70	-16.60	-41.40	-0.80	23.10
10.25	-10	4	2	-17.20	-38.90	-16.20	-46.10	-1.00	22.70
10.25	-10	4	4	-17.20	-37.50	-16.00	-41.00	-1.20	21.50
10.25	-10	4	6	-17.40	-37.60	-16.30	-41.70	-1.10	21.30
10.25	-10	4	8	-17.40	-37.00	-16.40	42.40	-1.00	21.20
10.25	-10	4	10	-17.40	-36.90	-16.60	-43.00	-0.80	20.30
10.25	-10	4	20	-17.40	-34.40	-16.50	-41.40	-0.90	17.90
10.25	-10	4	40	-17.40	-33.00	-17.20	-35.70	-0.20	15.80
10.25	-10	4	80	-17.40	-32.10	-17.90	-31.90	0.50	14.00
10.25	-10	4	160	-17.40	-27.90	-19.30	-30.40	1.90	8.60
10.25	-10	5	0.5	-17.40	-45.70	-16.50	-46.60	-0.90	29.20
10.25	-10	5	1	-17.40	-44.60	-16.10	-46.60	-1.30	28.50
10.25	-10	5	2	-17.20	-41.90	-15.70	-46.10	-1.50	26.20
10.25	-10	5	4	-17.20	-41.70	-16.10	-47.20	-1.10	25.60
10.25	-10	5	6	-17.40	-38.20	-15.70	-44.00	-1.70	22.50
10.25	-10	5	8	-17.40	-37.80	-15.80	-42.50	-1.60	22.00
10.25	-10	5	10	-17.40	-37.30	-16.10	-41.00	-1.30	21.20
10.25	-10	5	20	-17.40	-36.30	-16.60	-37.90	-0.80	19.70
10.25	-10	5	40	-17.40	-35.50	-17.40	32.90	0.00	15.50
10.25	-10	5	80	-17.40	-31.40	-18.00	31.20	0.60	13.20
10.25	-10	5	160	-17.40	-28.10	-19.00	31.10	1.60	9.10
10.25	-10	6	0.5	-17.40	-50.40	-16.00	-51.80	-1.40	34.60
10.25	-10	6	1	-17.40	-49.30	-15.80	-51.10	-1.60	33.50
10.25	-10	6	2	-17.20	-45.60	-15.60	-48.40	-1.60	30.00
10.25	-10	6	4	-17.20	-40.70	-15.40	-43.40	-1.80	25.30
10.25	-10	6	6	-17.40	-40.30	-15.60	-41.70	-1.80	24.70
10.25	-10	6	8	-17.40	-40.00	-15.80	-40.40	-1.60	24.20
10.25	-10	6	10	-17.40	-39.50	-16.00	-38.10	-1.40	22.10
10.25	-10	6	20	-17.40	-39.00	-16.50	-35.00	-0.90	18.50
10.25	-10	6	40	-17.40	-34.50	-17.10	-32.10	-0.30	15.00
10.25	-10	6	80	-17.40	-31.20	-17.50	-31.60	0.10	13.70
10.25	-10	6	160	-17.40	-27.70	-18.60	-30.70	1.20	9.10

* = = > or any other upper harmonic component with amplitude higher than P_{N-1}

Table 16. EFFECT OF CARRIER FREQUENCY ON T.L. AND S.R.

P_{in} (dBm)	f_T (kHz)	Bit	f_{in} (GHz)	P_0 (dBm)	P_{-N-1} (dBm)	P_1 (dBm)	P^*_{N-1} (dBm)	*T.L.* (dB)	*S.R.* (dB)
-10	10	1	9.5	-16.50	-20.00	-20.00	-29.40	3.50	0.00
			10.0	-16.90	-20.30	-20.30	-29.70	3.40	0.00
			10.25	-17.40	-20.80	-20.80	-30.20	3.40	0.00
			10.5	-17.10	-20.50	-20.50	-29.90	3.40	0.00
			11.0	-18.20	-21.30	-21.30	-30.70	3.10	0.00
-10	10	2	9.5	-16.50	-25.70	-16.70	-30.80	0.20	9.00
			10.0	-16.90	-25.80	-16.90	-30.90	0.00	8.90
			10.25	-17.40	-26.70	-17.60	-31.70	0.20	9.10
			10.5	-17.10	-26.70	-17.70	-31.70	0.60	9.00
			11.0	-18.20	-27.40	-18.80	-32.90	0.60	8.60
-10	10	3	9.5	-16.50	-31.50	-16.00	-36.00	-0.50	15.50
			10.0	-16.90	-31.70	-16.20	-36.20	-0.70	15.50
			10.25	-17.40	-32.20	-16.70	-36.60	-0.70	15.50
			10.5	-17.10	-32.60	-17.10	-37.00	0.00	15.50
			11.0	-18.20	-33.20	-18.10	-38.10	-0.10	15.10
-10	10	4	9.5	-16.50	-34.90	-15.40	-41.00	-1.10	19.50
			10.0	-16.90	-34.90	-15.60	-41.40	-1.30	19.30
			10.25	-17.40	-36.90	-16.60	-43.00	-0.80	20.30
			10.5	-17.10	-35.50	-16.50	-42.00	-0.60	19.00
			11.0	-18.20	-38.10	-18.20	-42.00	0.00	19.90
-10	10	5	9.5	-16.50	-37.90	-15.60	-43.20	-0.90	22.30
			10.0	-16.90	-37.40	-15.60	-40.80	-1.30	21.80
			10.25	-17.40	-37.30	-16.10	-41.00	-1.30	21.20
			10.5	-17.10	-38.30	-16.60	-41.70	-0.50	21.70
			11.0	-18.20	-40.40	-18.10	-41.80	-0.10	22.30
-10	10	6	9.5	-16.50	-40.40	-15.40	-38.90	-1.10	25.00
			10.0	-16.90	-39.70	-15.40	-39.30	-1.40	24.30
			10.25	-17.40	-39.50	-16.00	-38.10	-1.40	22.10
			10.5	-17.10	-40.40	-16.40	-39.20	-0.70	22.80
			11.0	-18.20	-41.10	-17.40	-39.20	-0.80	21.80

* = = · or any other upper harmonic component with amplitude higher than P_{N-1}

Table 17. EFFECT OF CARRIER INPUT POWER ON T.L. AND S.R.

f_{in} (GHz)	f_T (kHz)	Bit	P_{in} (dBm)	P_0 (dBm)	P_{-N-1} (dBm)	P_1 (dBm)	P^*_{N-1} (dBm)	*T.L.* (dB)	*S.R.* (dB)
10.25	10	1	-40	-47.10	-50.40	-50.40	-59.70	3.30	0.00
			-30	-36.70	-40.10	-40.10	-49.60	3.40	0.00
			-20	-26.80	-30.10	-30.10	-39.70	3.30	0.00
			-10	-17.40	-20.80	-20.80	-30.20	3.40	0.00
			0	-7.30	-10.60	-10.60	-20.20	3.30	0.00
10.25	10	2	-40	-47.10	-56.30	-47.40	-61.10	0.30	8.90
			-30	-36.70	-46.10	-37.00	-51.00	0.30	9.10
			-20	-26.80	-36.20	-27.00	-41.20	0.50	9.20
			-10	-17.40	-26.70	-17.60	-31.70	0.20	9.10
			0	-7.30	-16.70	-7.50	-21.70	0.20	9.20
10.25	10	3	-40	-47.10	-61.30	-46.50	-64.90	-0.6	14.80
			-30	-36.70	-51.50	-36.20	-55.80	-0.50	15.30
			-20	-26.80	-41.70	-26.30	-46.20	-0.50	15.40
			-10	-17.40	-32.20	-16.70	-36.60	-0.70	15.50
			0	-7.30	-22.20	-6.70	-26.80	-0.70	15.50
10.25	10	4	-40	-47.10	-65.40	-46.50	-70.00	-0.60	18.90
			-30	-36.70	-56.10	-36.10	-61.90	-0.60	20.00
			-20	-26.80	-46.40	-26.20	-52.30	-0.60	20.20
			-10	-17.40	-36.90	-16.60	-43.00	-0.80	20.30
			0	-7.30	-26.80	-6.60	-33.10	-0.70	20.20
10.25	10	5	-40	-47.10	-65.40	-45.80	-67.40	-1.30	19.60
			-30	-36.70	-56.30	-35.50	-60.4	-1.20	20.80
			-20	-26.80	-46.90	-25.60	-50.80	-1.20	21.30
			-10	-17.40	-37.30	-16.10	-41.00	-1.30	21.20
			0	-7.30	-27.40	-6.00	-31.30	-1.30	21.40
10.25	10	6	-40	-47.10	-66.30	-45.70	-65.90	-1.40	20.20
			-30	-36.70	-58.50	-35.40	-57.40	-1.30	22.00
			-20	-26.80	-49.40	-25.50	-48.30	-1.30	22.80
			-10	-17.40	-39.50	-16.00	-38.10	-1.40	22.10
			0	-7.30	-29.30	-5.80	-27.90	-1.50	22.10

* = = = or any other upper harmonic component with amplitude higher than P_{N-1}

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